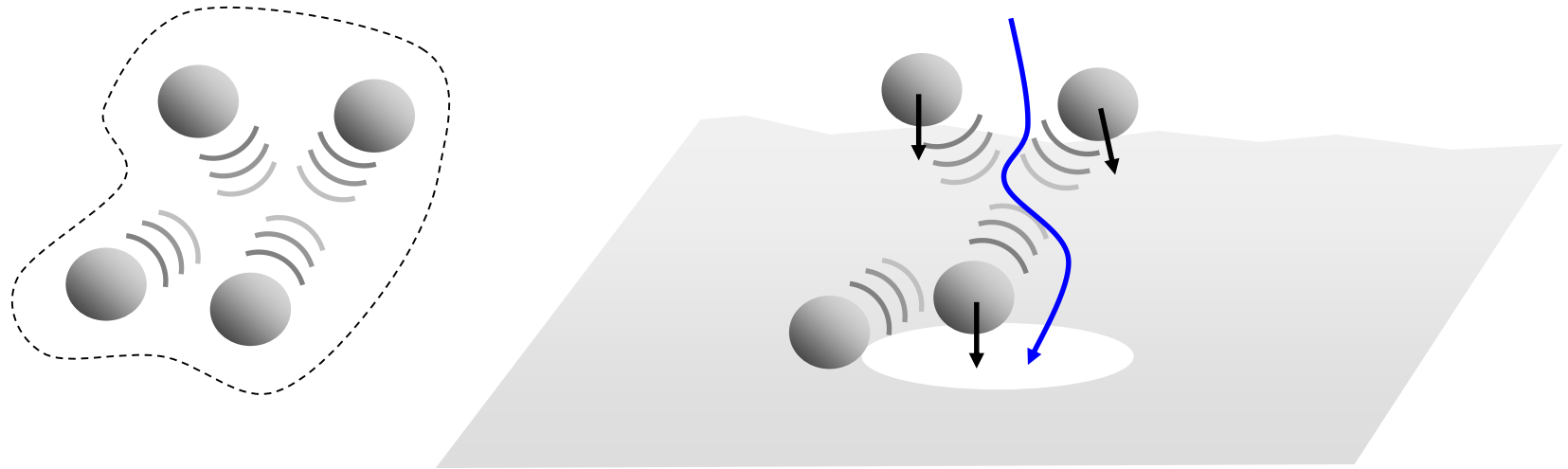


*From colloidal interactions ...*

*... to transport phenomena in processes*

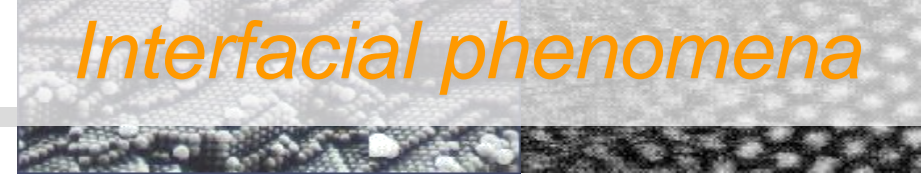


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*Université Paul Sabatier – Bat IIR1*



1

## Problematic

- How to model colloid transport in a process ?

## Introduction

- What are colloids ?
- Problematic

From causes....

## Electrostatic charges

- Origin
- Charge distribution at an interface (Gouy-Chapman theory)

## Interactions between interfaces

- van der Waals attraction
- Electrostatic repulsion
- Interparticles interactions (DLVO theory)

... to consequences.

## Electrokinetic phenomena

- Electrophoresis
- electro-osmosis
- Streaming potential
- Settling potential

## Aggregation

- Orthokinetic aggregation : slow or rapid ?
- Perikinetic Aggregation
- Population balance -> E. Climent course

## Transport phenomena

- Osmotic pressure
- mobility
- diffusion
- settling
- filtration ...

## Viscosity et rheology

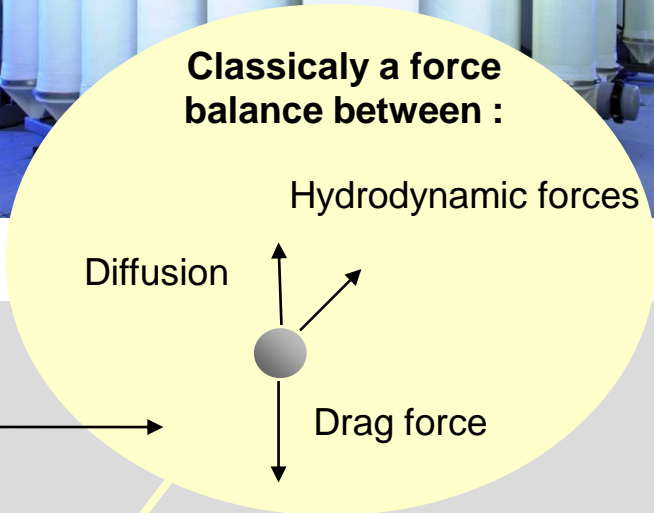
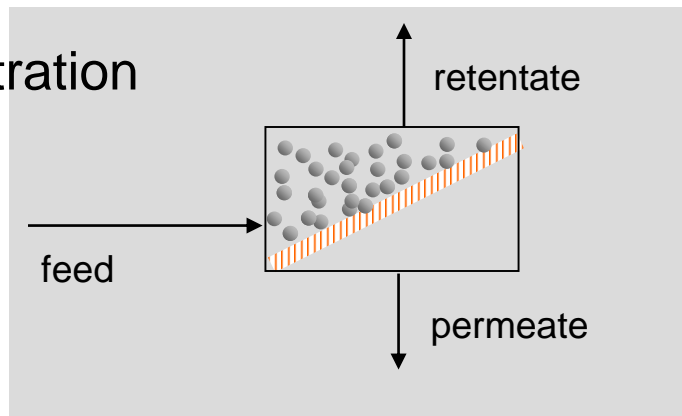
-> C. Xuereb course



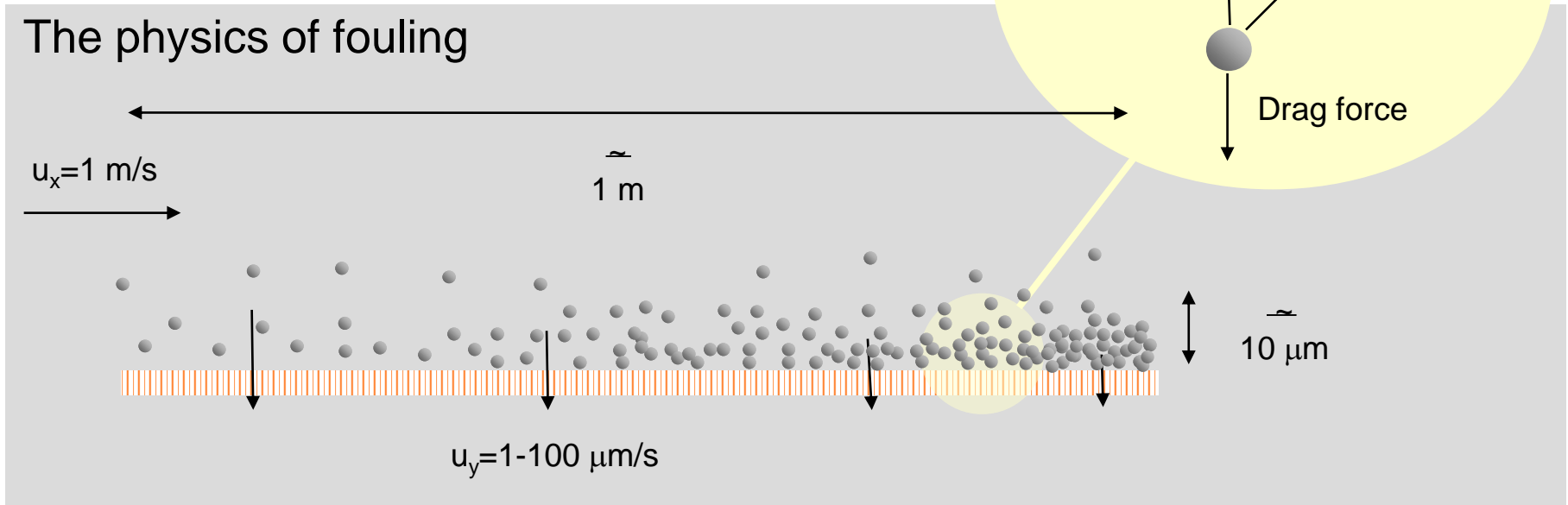
# The problem

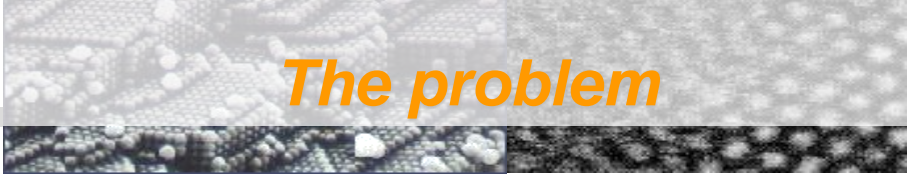
## A new kind of complexity in a process

### Example with filtration



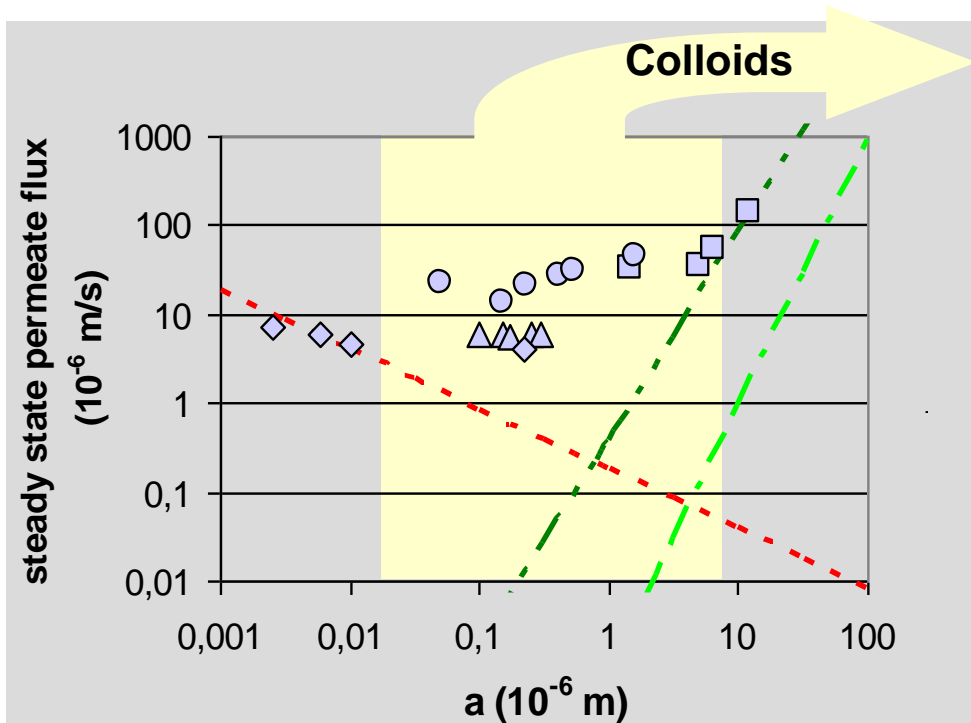
### The physics of fouling





The problem

A new kind of complexity in a process (2)



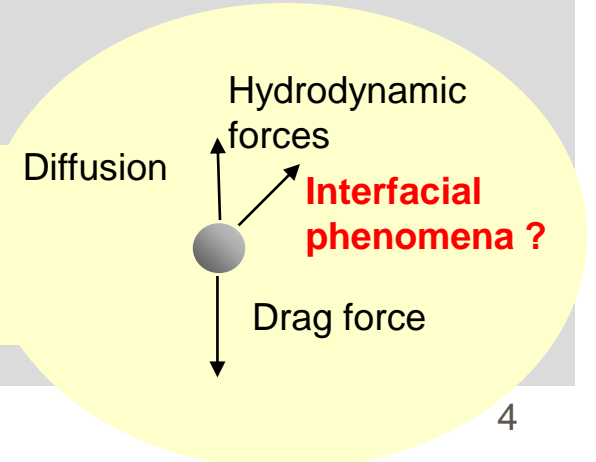
Permeation flux can not be predicted by :

- - - Diffusion
- . - Lateral migration
- . . . Shear induced diffusion

**Hydrodynamic phenomena**

« Colloid flux paradox »

**How to model colloids transport in a process ?**





# What are colloids?

## Solutions

- *Brownian diffusion*
- *Stable solubilised state*

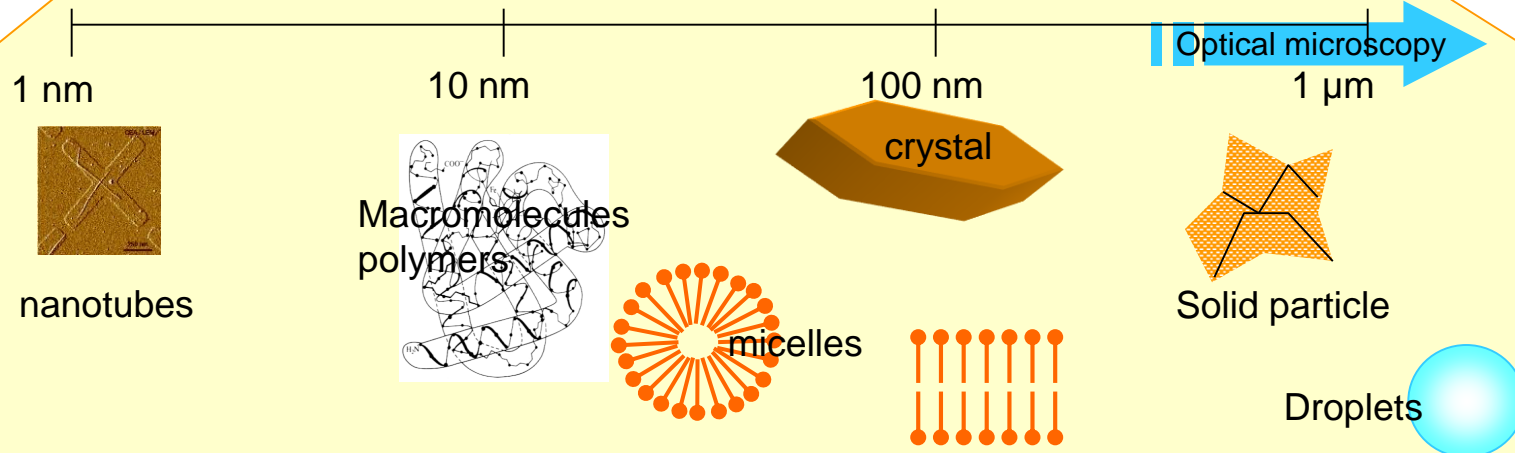
## Colloidal dispersions

- *Brownian diffusion > Gravity*
- *Dispersed state = metastable*

## Suspensions

- *Settling under gravity*
- *Suspended by mixing to be dispersed*

Interfacial area= 100 m<sup>2</sup>/g



According IUPAC\* :

*the supramolecular entities whose extension in at least one spatial direction lies between 1 nm and 1 μm*

\* International Union of Pure and Applied Chemistry



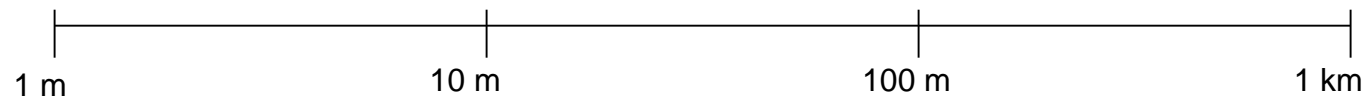
# Introduction

## Un univers de taille et de forme !

L'échelle des colloïdes ...



... à l'échelle humaine



## Une grande variété de taille et de volume

Il y a 1 milliards de  $\text{nm}^3$  dans  $1 \mu\text{m}^3$

## What are colloids?

Media	Particle	Type	Natural	Technique
liquid	solid	sol	superficial water	ink, paint
liquid	liquid	emulsion	milk	oil
liquid	gas	foam	sparkling water	Fire extinguishers
Gas	solid	aerosol	smoke	Pharmaceutical to inhale
Gas	liquid	aerosol	cloud	insecticide
solid	solid	alloy	wood, bone	Composite materials
solid	liquid	porous media	petrol, opal	Polymeric membrane
solid	gas	solid foam	snakestone	zeolite

Colloids are found in a **lot of processes**.

## What are colloids?

A small volume with a large interfacial area

$$\frac{A}{V} = \frac{\text{surface des particules}}{\text{volume de la suspension}} = \overbrace{\frac{4\pi a^2}{\frac{4}{3}\pi a^3} \phi}^{\text{spheres}} = \frac{6\phi}{d_p}$$



$d_p$	area
1 $\mu\text{m}$	300 $\text{m}^2$
100 nm	3000 $\text{m}^2$
10 nm	3 $10^4$ $\text{m}^2$
1 nm	3 $10^5$ $\text{m}^2$

Colloids are controlled by the **interfacial properties and the interactions between interfaces** (rather than by the chemical composition).



## Perpétuellement en mouvement

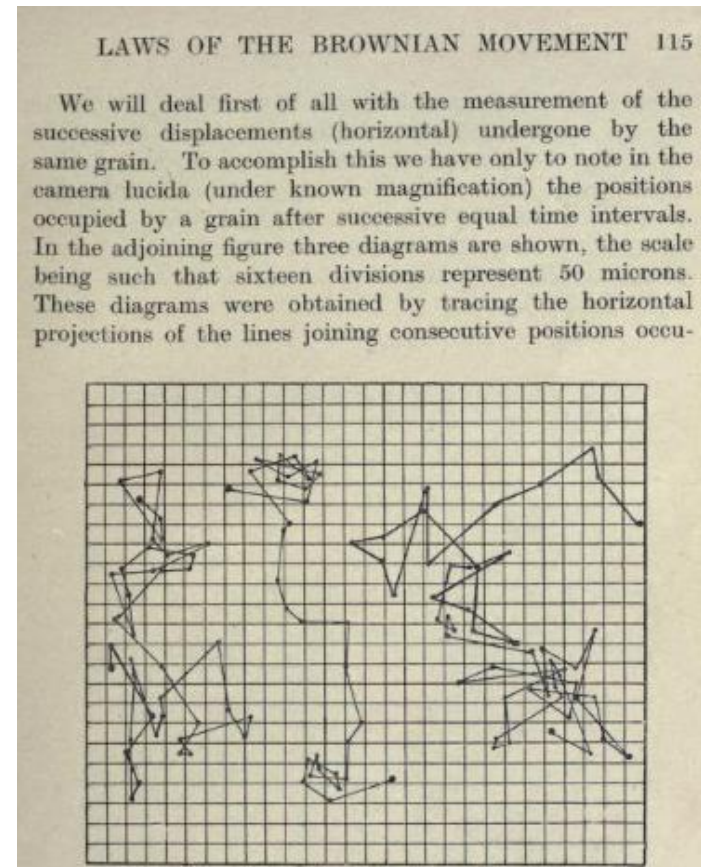
Un colloïde : ça diffuse

$$D = \frac{kT}{6\pi\mu R}$$

**2R**      *distance moyenne  
parcourue en 1 min*

1 $\mu\text{m}$	5 $\mu\text{m}$
100 nm	16 $\mu\text{m}$
10 nm	50 $\mu\text{m}$
1 nm	0.16 mm

$$d = \sqrt{Dt}$$



Mouvement de colloïdes de 0.53  $\mu\text{m}$  de rayon observé sous microscope à intervalle de 30 seconds (taille d'un carré 3.2  $\mu\text{m}$ )  
Jean Baptiste Perrin, Atoms, 1914

## Colloids and soft matter

Soft matter  
 Systèmes moléculaires organisés  
 Objets fragiles  
 Matière mal condensée

Entities that **interact weakly** (compared with the chemical reaction)

But on **important distance range** (the colloid size increased by a factor of 10)

The properties are then controlled by these weak interactions:

a small change in these interactions (low energy is required)

can lead to substantial modification.

Physique de la matière  
 Mécanique  
 Physico-Chimie  
 Physique statistique

*... on peut transformer la matière avec des actions extérieures faibles ...*

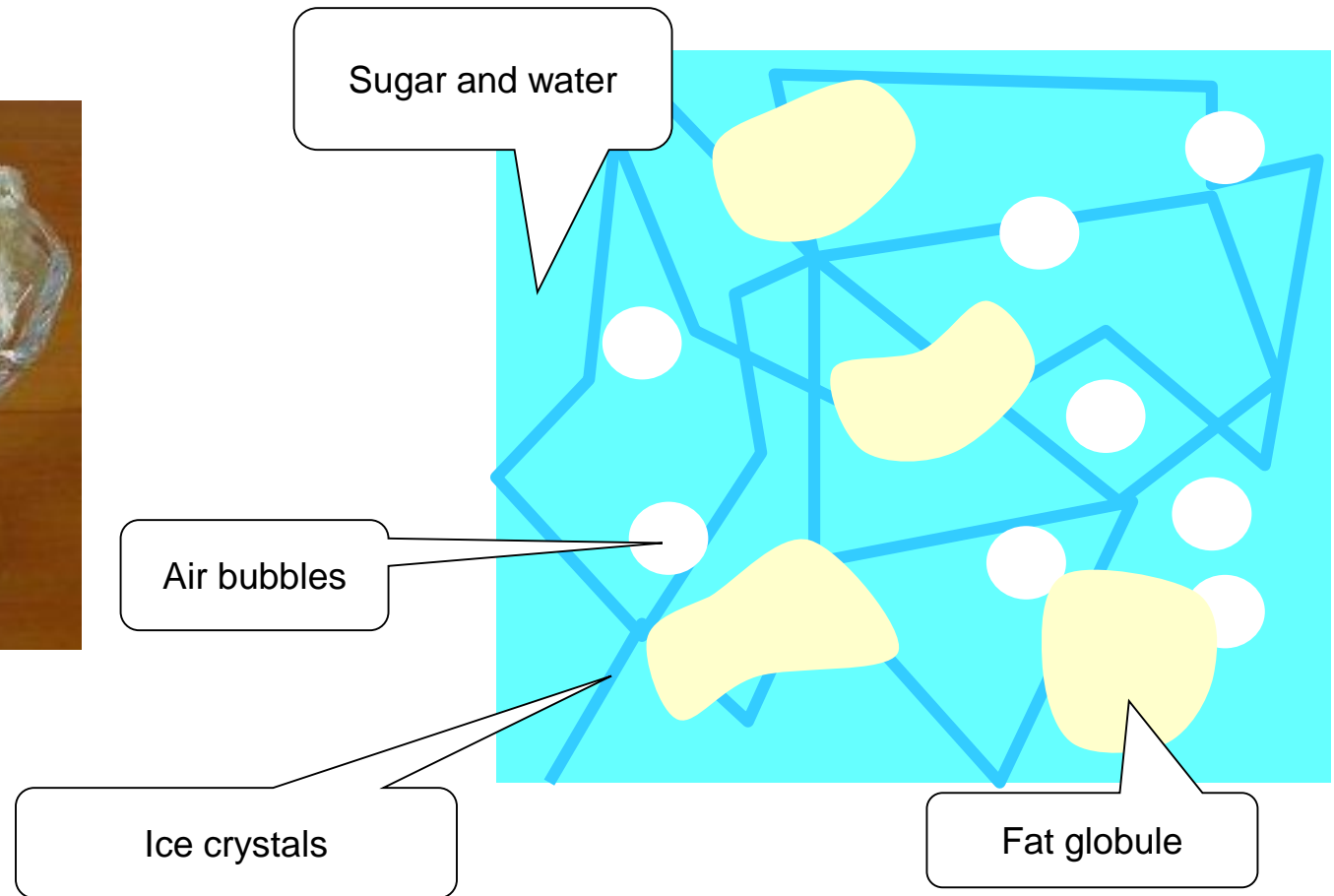
*Voilà la définition centrale de la matière molle.*

Pierre Gilles de Gennes et Jacques Badoz, Les objets fragiles



# Colloid and soft matter

## Ice cream



## Problematic

*Les interactions (...) entraînent un accroissement de complexité source de l'émergence de performances inattendues.*

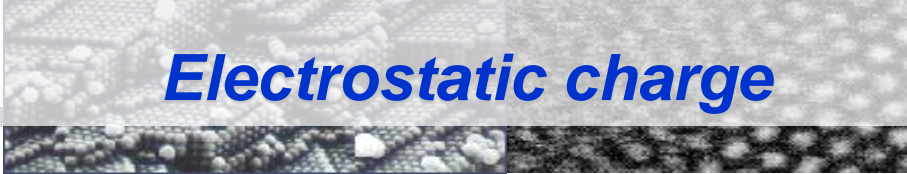
Albert Jacquard, L'équation du nénuphar

*... Important areas of physical chemistry such as interfacial phenomena, colloids, clusters and, more generally, De Gennes "soft matter" should be revisited using the system approach and chemical engineering methods.*

Jacques Villiermaux, Future challenges for basic research in chemical engineering  
Chemical Engineering Science, 48 (1993)

*...mais totalement ignorante de la " matière molle ". Nous souffrons en France d'une certaine spécialisation du génie chimique. On n'y trouve pas toujours la variété de culture exhibée par les départements américains de Chemical Engineering.*

Pierre Gilles de Gennes, Chimistes et physiciens : synergies et lacunes  
L'actualité chimique, 258 (2005)



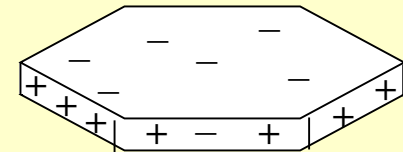
# Electrostatic charge

The main part of macromolecules and particles are charged:

## Structural origin

Substitution of ions :  $\text{Si}^{4+}$  by  $\text{Al}^{3+}$  or  $\text{Mg}^{2+}$

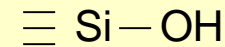
Clay charges



## Ionisable group at the surface

Amphoteric group

Silica  
Protein  
Oxide

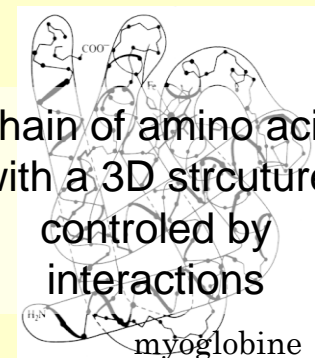


## Selective ionic adsorption

Adsorption of anions (less hydrated)

AgI

Chain of amino acid  
with a 3D structure  
controlled by  
interactions



Co-ions and counter-ions distribution near a charged surface:

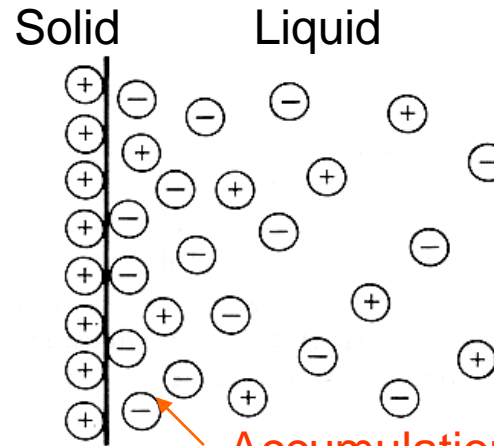
*The electrostatic double layer*

Distribution of electrostatic potential :

Poisson equation  
(charge distribution -> electrostatic potential)

$$\frac{c_i}{c_{i0}} = \exp\left(\frac{-z_i e \psi}{k_B T}\right)$$

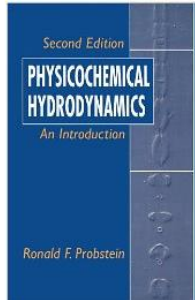
Boltzmann equation  
(electrostatic potential → ions distribution)



Accumulation of counter-ions near the interface

$$\nabla^2 \psi = -\frac{\rho'}{\epsilon}$$

$$\rho' = F \sum_i z_i c_i$$





## Electrostatic potential distribution (Gouy-Chapman theory)

Assumptions: - plane surface

- Debye-Hückel approximation  $z_i e \psi \ll k_B T$

$$\psi < 25,7 \text{ mV}$$

$$\psi = \psi_w e^{-\frac{x}{\lambda_D}} \quad \text{(I)}$$

Exact solution :

$$\tanh(z\hat{\psi} / 4) = \tanh(z\hat{\psi}_0 / 4) \exp(-\kappa_D x)$$

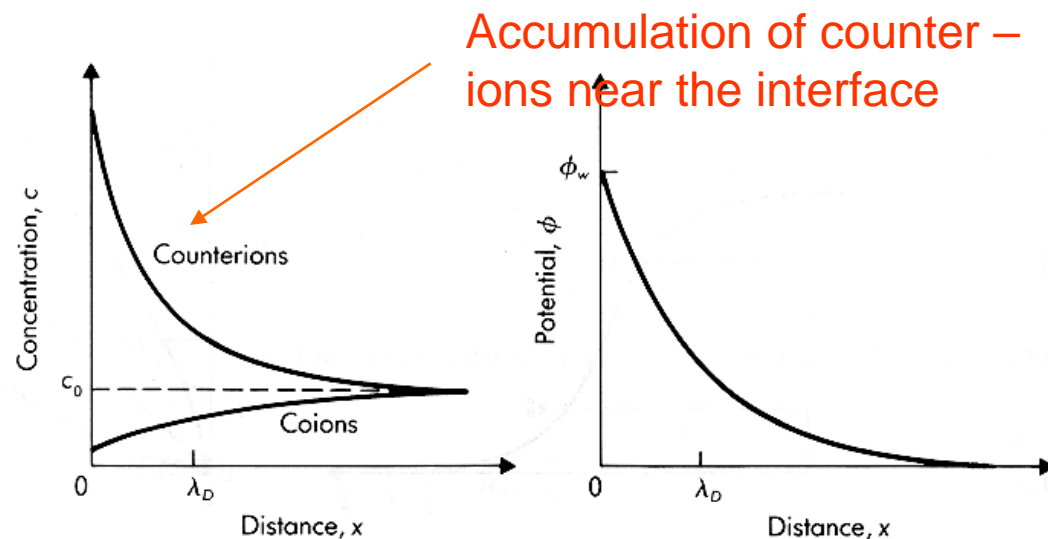
avec  $\hat{\psi} = e\psi / kT$

### With the Debye length

$$\lambda_D = \sqrt{\frac{\epsilon RT}{2F^2 \sum z_i^2 c_i}} = \frac{3.07 \cdot 10^{-10}}{\sqrt{I}} \quad \begin{matrix} \text{m} \\ \text{mol/l} \end{matrix}$$

**Force ionique :**  $I = \frac{1}{2} \sum_i z_i^2 c_i$

$$\lambda_D = \frac{1}{\kappa_D}$$



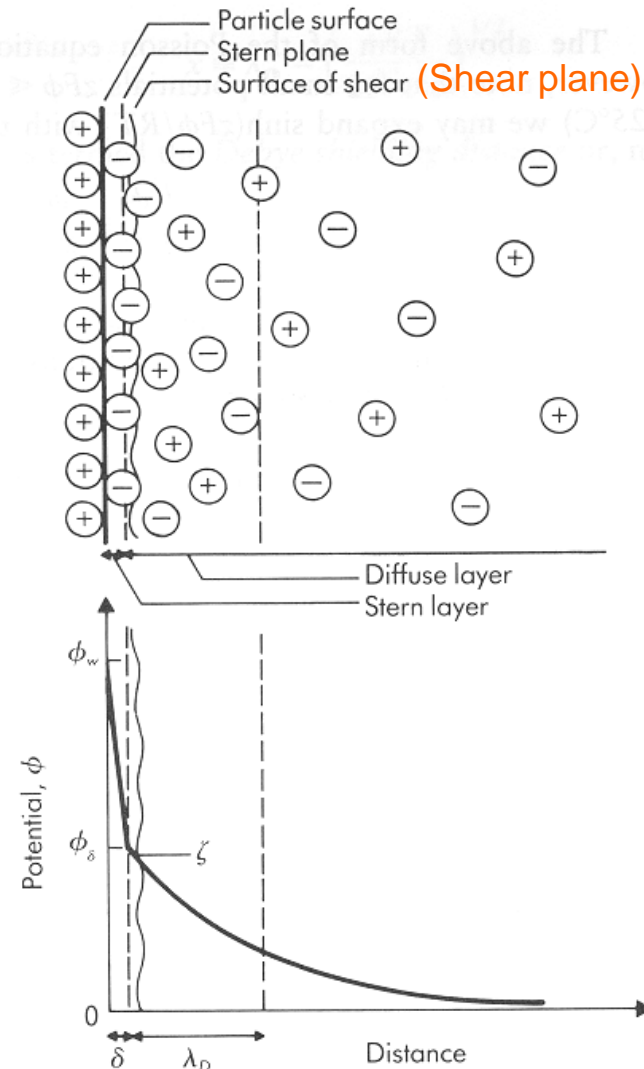
## Electrostatic double layer et Stern internal plane

The **zéta potential ( $\zeta$ )** is defined  
as the **potential at the shear plane.**  
(experimentally reachable)

As  $\delta \ll \lambda_D$  the equation (I)

can be written with

$$\psi_w = \zeta$$



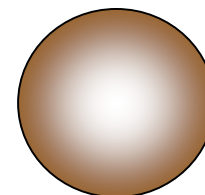


## Electrostatic double layer around a sphere

*Poisson-Boltzmann* equation around a sphere

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\psi}{dr} \right) = -\frac{F}{\varepsilon} \sum_i z_i c_{i0} \exp\left( \frac{-z_i e\psi}{k_B T} \right)$$

No analytical solution !



With the *Debye-Hückel* approximation :

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\psi}{dr} \right) = \kappa_D^2 \psi$$

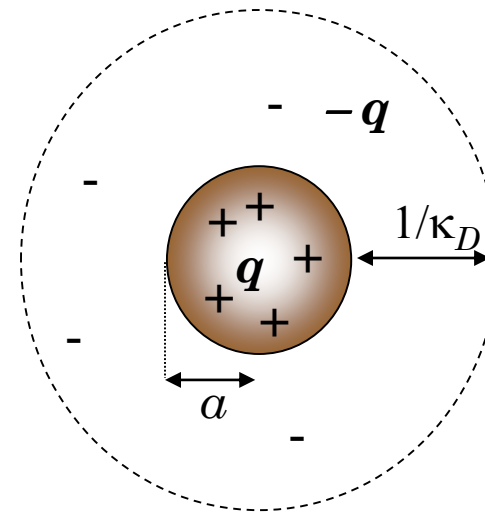
$$\psi = \psi_w a \frac{\exp[-\kappa_D (r - a)]}{r}$$

## Electrostatic double layer around a sphere

### Electroneutrality

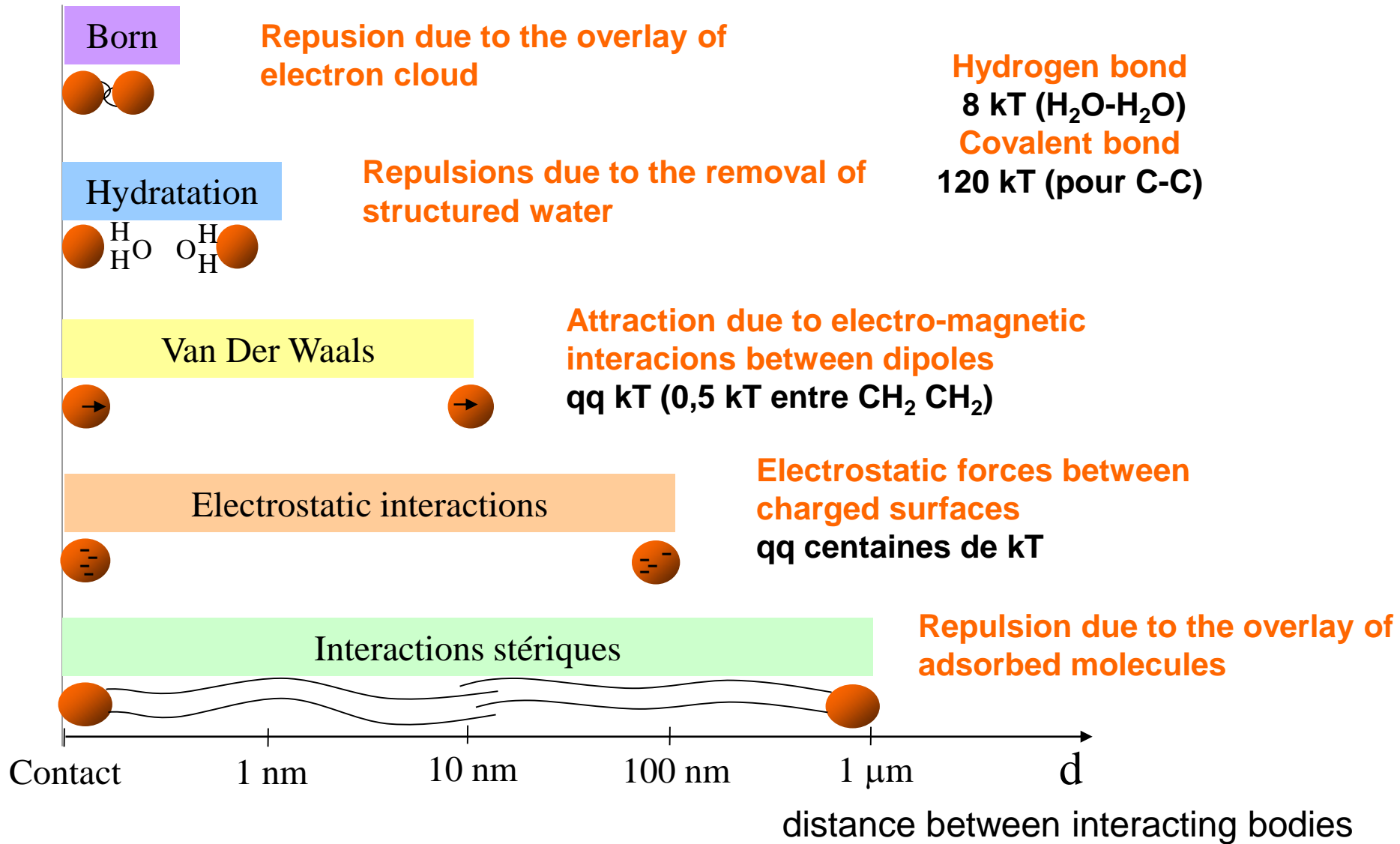
charge on the particule = charge in the double layer

$$\begin{aligned}
 q &= -\int_a^{\infty} 4\pi r^2 \rho' dr \\
 &= 4\pi\epsilon\kappa_D^2 \int_a^{\infty} r^2 \psi dr \\
 &= 4\pi\epsilon a(1 + \kappa_D a)\psi_0
 \end{aligned}$$



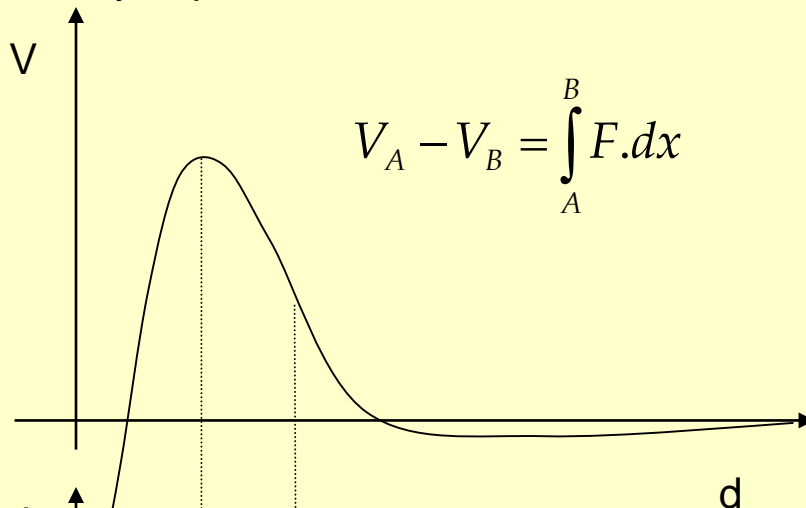
If  $a$  is the radius until the shear plane:

$$q = 4\pi\epsilon a(1 + \kappa_D a)\zeta$$



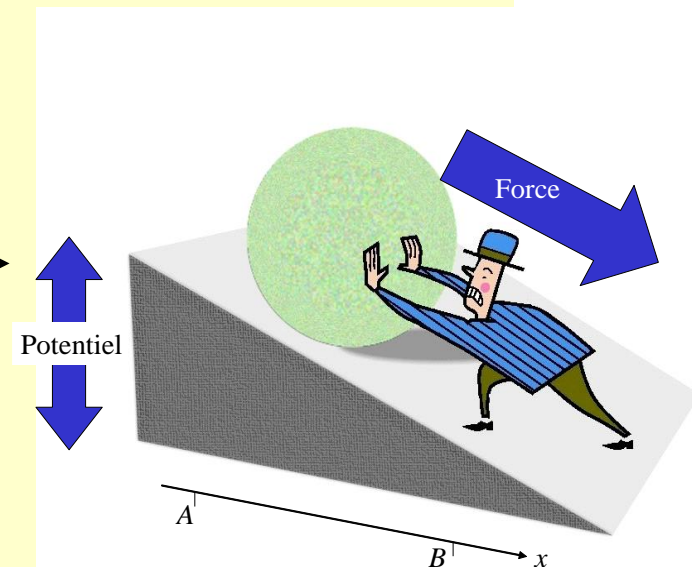
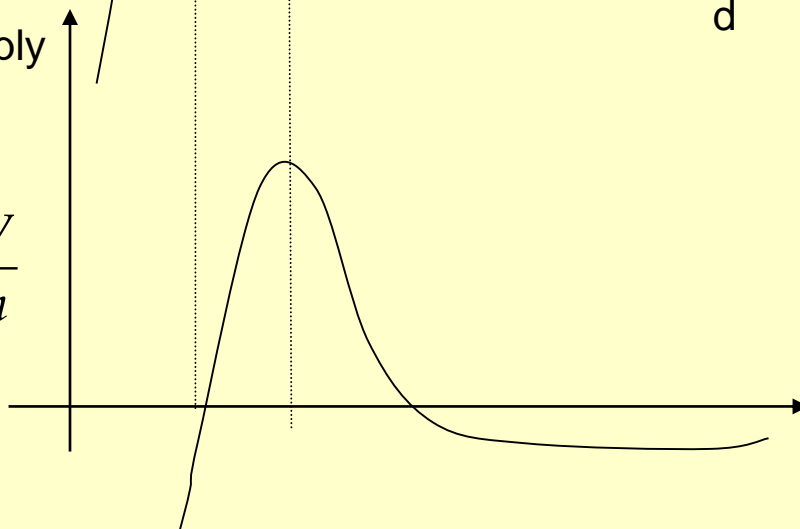
Definition : the potentiel energy of interaction

Energy,  $V$ , required to approach two surfaces infinitely separated until a distance  $d$



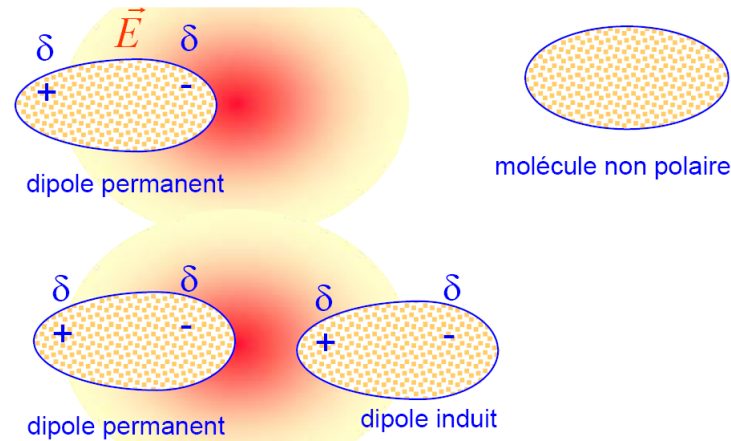
Force,  $F$ , to supply to approach the two surfaces

$$F = -\frac{dV}{dh}$$



Van der Waals interactions

**Attractive force** between permanent dipoles (KEESOM 1915)  
 permanent dipole–induced dipole (DEBYE 1921) } Polar forces



induced dipoles (LONDON 1930)

} Dispersive force

Consequences

- between atoms
  - gap to ideal gas law
- between molecules
  - superfical tension
- between macromolecules or particles
  - attractive potential énergie

## Attractive potential of *van der Waals*

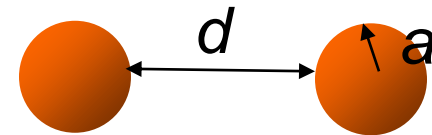
→ between two infinite plate :

$$V_A = -\frac{A}{12\pi} d^{-2}$$

$J/m^2$



→ between two identical spheres



$$V_A = -\frac{A}{6} \left[ \frac{2a^2}{d^2 + 4ad} + \frac{2a^2}{d^2 + 4ad + 4a^2} + \ln \left( \frac{2a^2}{d^2 + 4ad + 4a^2} \right) \right]$$

Si  $a \gg d$        $V_A = -\frac{Aa}{12d}$        $J$

→ between two spheres with different radius :

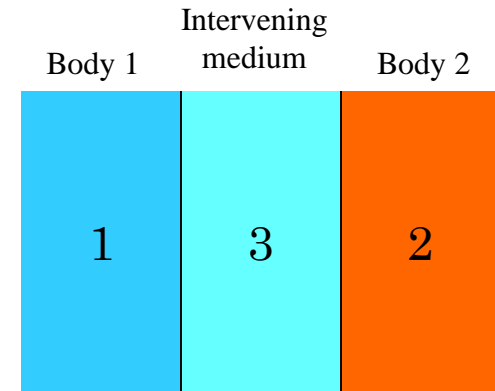
Si  $a_1 \gg d$       et       $a_2 \gg d$        $V_A = -\frac{Aa_1a_2}{6d(a_1 + a_2)}$        $J$

## Hamaker constant

$$A_{132} \approx \pm \sqrt{A_{131} A_{232}}$$

If the intervening medium is vacuum:

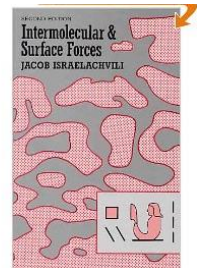
$$A_{12} \approx \sqrt{A_{11} A_{22}}$$



$$A_{131} \approx A_{313} \approx A_{11} + A_{33} - 2A_{13} \approx \left( \sqrt{A_{11}} - \sqrt{A_{33}} \right)^2$$

$$A_{132} \approx \left( \sqrt{A_{11}} - \sqrt{A_{33}} \right) \left( \sqrt{A_{22}} - \sqrt{A_{33}} \right)$$

Israelachvili  
1991



A  
Hamaker constant  
 $10^{-19} - 10^{-20}$  J

AN : Estimate A for  
polystyrene / eau / or

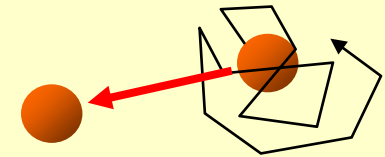
Materials	A / $10^{-20}$ J	
	Vacuum	Water
Polystyrene	7.9	1.3
Hexadecane	5.4	-
Gold	40	30
Silver	50	40
Al <sub>2</sub> O <sub>3</sub>	16.75	4.44
Copper	40	30
Water	4.0	-

**VdW energy and force ranges:**

200 nm spheres of polystyrene (latex) dispersed in water

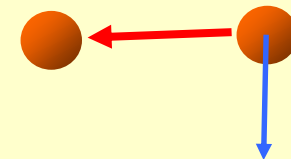
VdW compared with the diffusion :

d	1 nm	10 nm
$V_{A\text{ vdw}}/kT$	-26	-2.6



VdW compared with the drag force  
(induced by a velocity of 1 mm/s) :

d	10 nm
$F_{A\text{ vdw}}$	$-1.1 \cdot 10^{-12} \text{ N}$
$F_{\text{Drag}}$	$1.9 \cdot 10^{-12} \text{ N}$



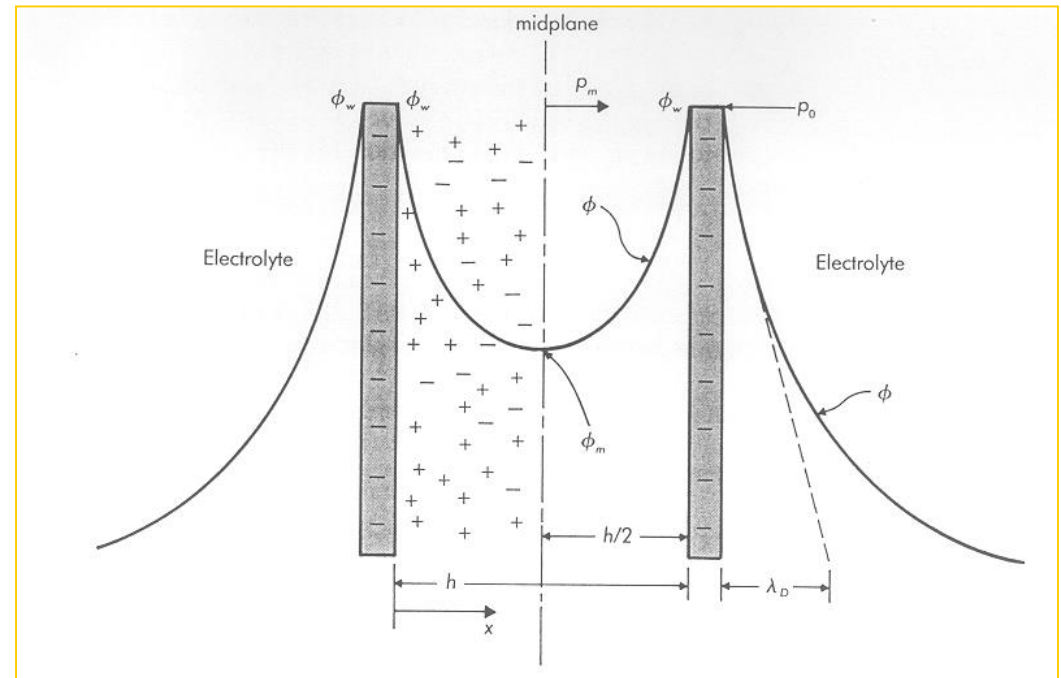


## Electrostatic forces between two charged surfaces

Double layer  
superposition



Repulsion  
between the  
surfaces



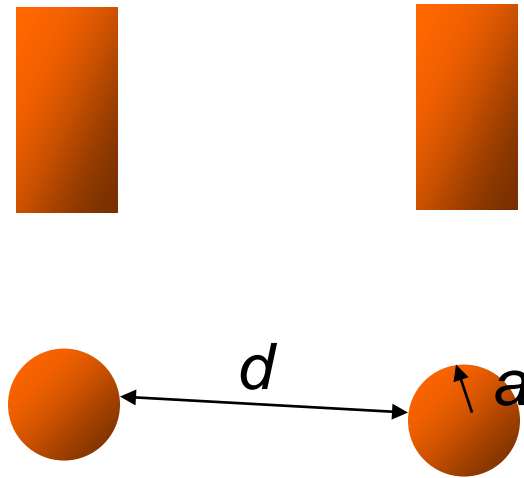
This force is often called ionic repulsion to underline its difference with a pure Coulombian repulsion

## Repulsion potentiel energy

for a symmetric electrolyte z:z

Between two plates:

$$V_R = \frac{64n_0k_B T}{\kappa_D} \Upsilon_0^2 \exp(-\kappa_D d)$$



Between two spheres:

$$V_R = \frac{64\pi a n_0 k_B T}{\kappa_D^2} \Upsilon_0^2 \exp(-\kappa_D d)$$

$$\Upsilon_0 = \tanh\left(\frac{z\psi_0 e}{4k_B T}\right)$$

**DLVO theory:** *Deryaguine, Landau* (1941), *Verwey, Overbeek* (1948)

Total potentiel energy of interaction,  $V$  :

Electrostatic repulsion

van der Waals attraction

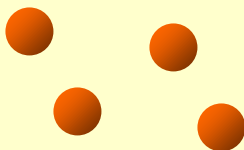
$$\begin{aligned}
 V &= V_R + V_A \\
 \text{Between two spheres :} &= \frac{64\pi a n_0 k_B T}{\kappa_D^2} \gamma_0^2 \exp(-\kappa_D d) - \frac{Aa}{12d}
 \end{aligned}$$



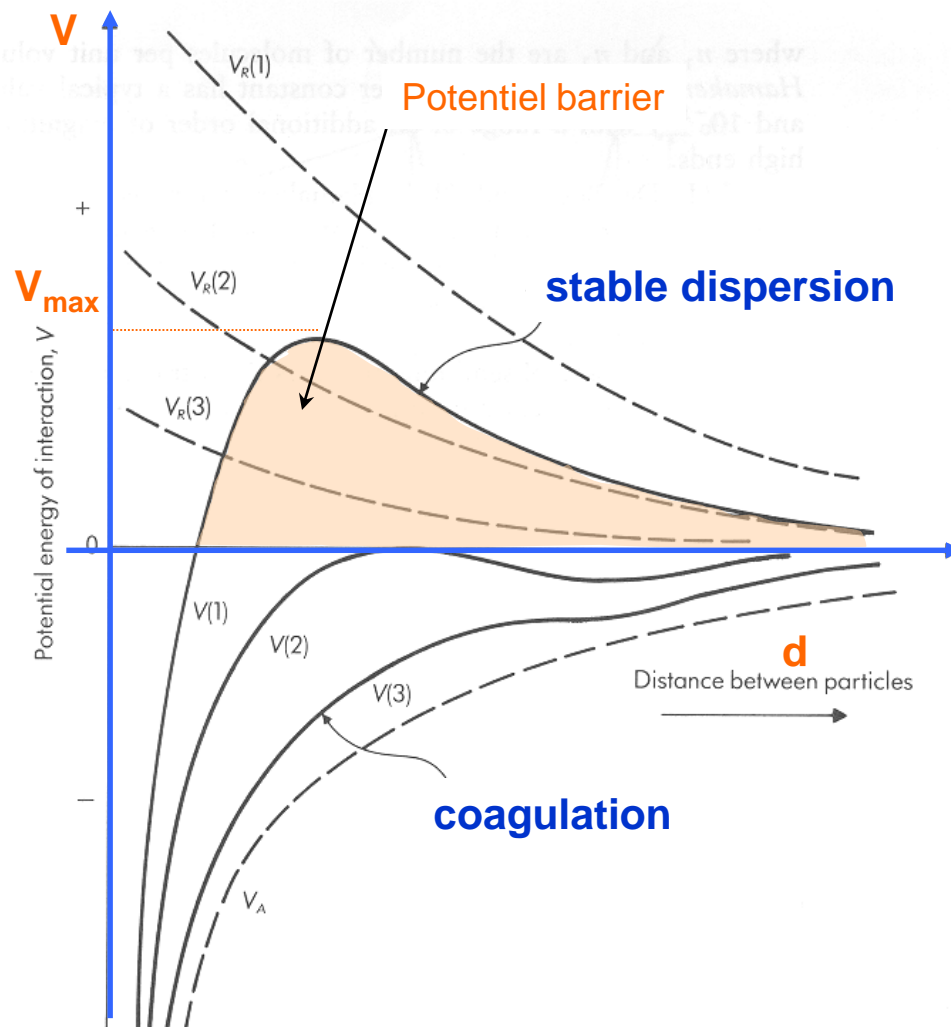
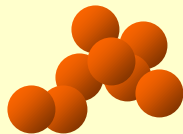
- The estimation of  $A$  is imprecise.
- The zêta potential gives a minimal value of  $\psi_0$
- The ionic strength control the value of  $\kappa_D$ .

## DLVO theory and colloids stability

if  $V_{\max} > kT$   
stable dispersion



if  $V_{\max} < kT$   
coagulation





## Critical Concentration in electrolyte for the Coagulation (c.c.c.)

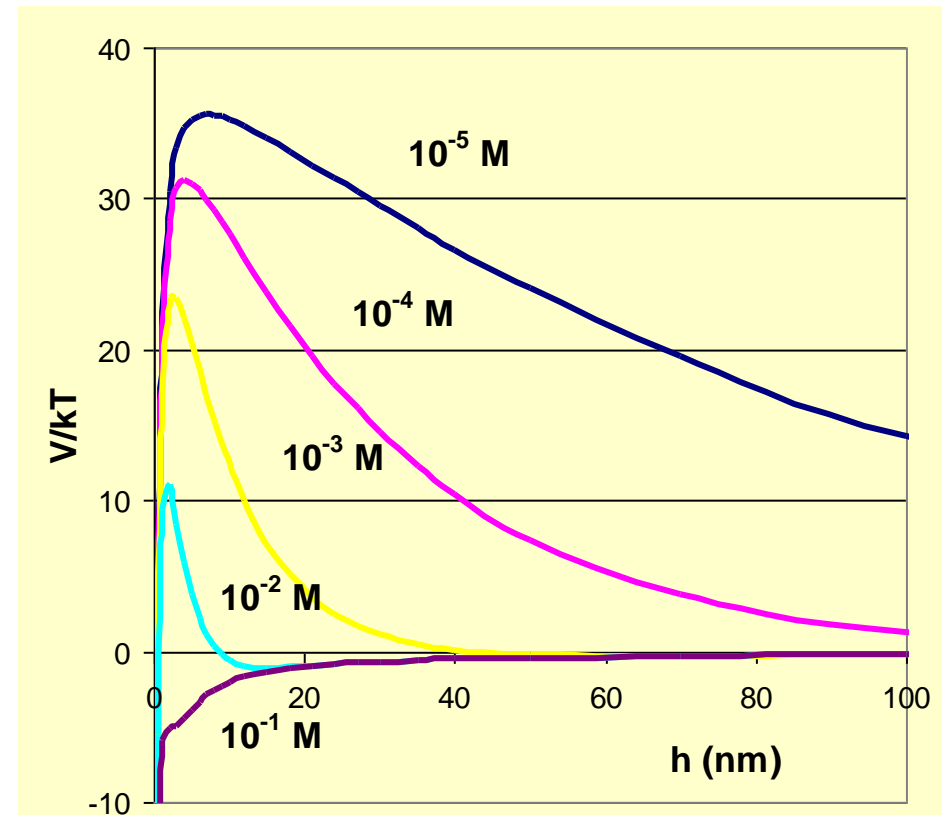
Schulze-Hardy rule

$$c_{crit} \text{ tel que } \left. \frac{dV}{dh} \right|_{h_{max}} = 0 \text{ et } V(h_{max})=0$$

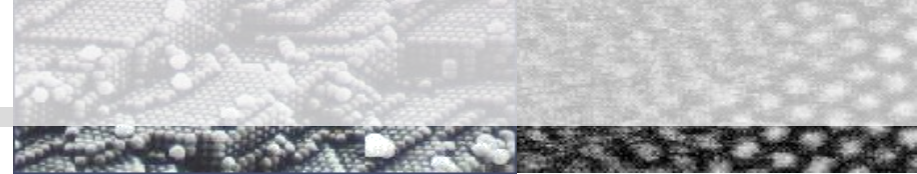
$$C_{crit} = 3,8 \cdot 10^{-36} \frac{\gamma^4}{A^2 z^6} \text{ mol/m}^3$$

Variation of the c.c.c. with the valency<sup>6</sup>

Na <sup>+</sup>	Ca <sup>2+</sup>	Al <sup>3+</sup>
100	1,56	0,137



Potential energy of interaction as a function of the body-body distance for 200 nm spheres with a zeta potential of 20 mV ( $A=1 \cdot 10^{-20}$  J)



```

k=1.38064852e-23 #m2 kg s-2 K-1
T=298
avo=6.02214076e23
kT=k*T
e=1.6e-19
#colloids
a=1e-7
A=1e-20
zeta=-0.02
#solution
c0=[0.00001,0.0001,0.001, 0.01,0.1] #mol/L
z=1.
#allocation variable
maxi=np.zeros(len(c0))
print ('c0 \t V_max')

#Fonctions pour le calcul de VR -repulsion- et VA -attraction-
def VR(d,zeta,c):
    I=z**2*c
    n0=c*1e3*avo
    lamD=3.07e-10/np.sqrt(I)
    gam0=np.tanh(z*zeta*e/(4*kT))
    return 64*np.pi*a*n0*kT*(lamD**2)*(gam0**2)*np.exp(-d/lamD)
def VA(d):
    return -A*a/(12*d)

```

## Stability, instability and metastability

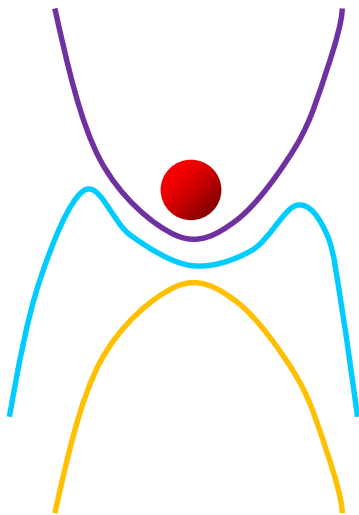
### **Stable state** (at equilibrium)

Pure liquid

Solution of hydrophilic or ionic molecules

Solution of hydrophobic molecules

Solution and association of amphiphilic molecule



### **Metastable state** (colloidal state)

(the evolution towards the equilibrium is blocked)

Solid/liquid dispersion

emulsion

gel

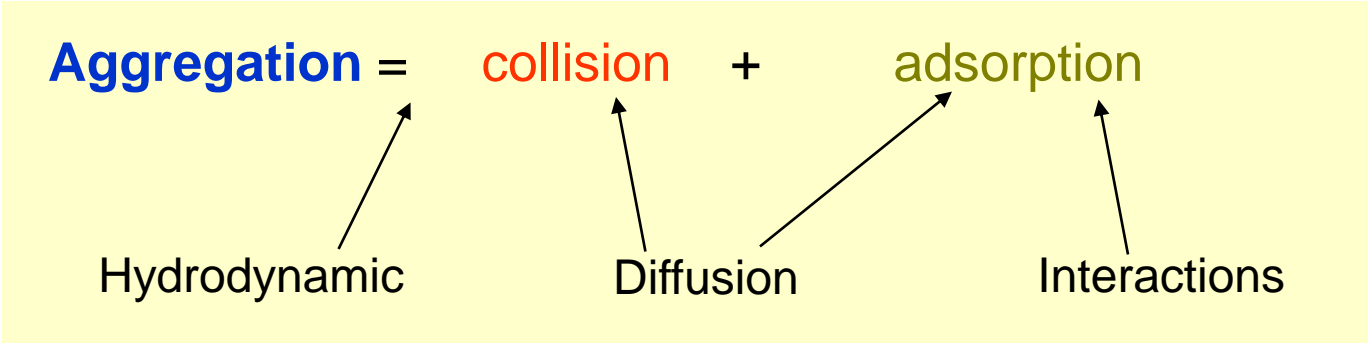
### **Instable state**

Immiscible solvent



# Aggregation

## Aggregation mode ?



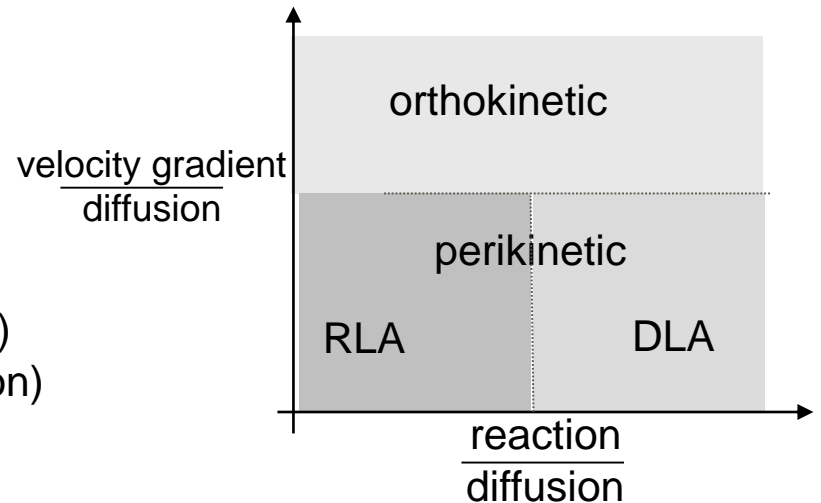
The Aggregation is controled by :

the **collision mechanisms** :

- Brownian diffusion (perikinetic aggregation)
- Velocity gradient (orthokinetic aggregation)

the **adsorption** :

- Brownian diffusion (Diffusion Limited Agregation)
- Surface interactions (Reaction Limited Agregation)





## Aggregation kinetics

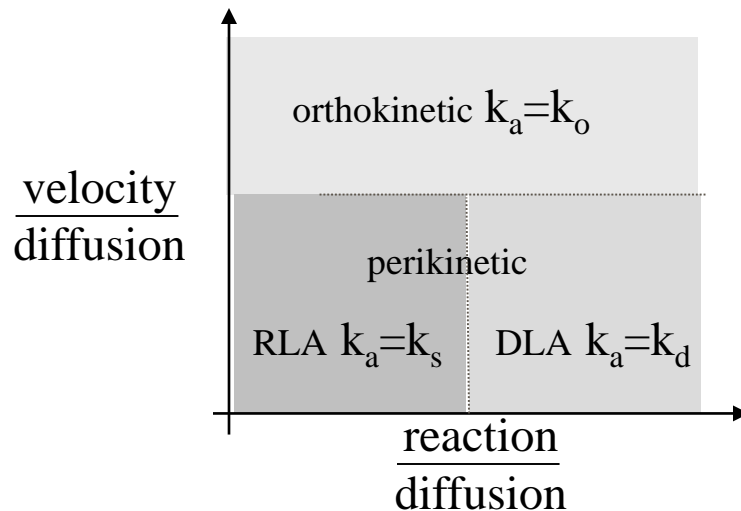
Number of interparticle collisions

$$= k_a n^2$$

order of reaction : 2

Kinetic of particle loss  
(n particles per unit volume)

$$\frac{dn}{dt} = -k_a n^2$$



$$n = \frac{n_0}{1 + k_a n_0 t}$$

initial particle size

$$n_0 = \frac{\phi}{\frac{4}{3}\pi\alpha^3}$$

## Perikinetic aggregation

$$k_s = \frac{8\pi Da}{W} = \left(\frac{1}{W}\right) k_D$$

constant for Brownian aggregation

collision efficiency

stability ratio

$$W = 2a \int_{2a}^{\infty} \exp(V / k_B T) r^{-2} dr$$

$$= 2a \int_{2a}^{\infty} \exp(V / k_B T) \frac{dh}{(h + 2a)^2}$$

$$k_D = \frac{4kT}{3\mu} = 6.10^{-18} m^3 .s^{-1}$$

constant for rapid aggregation (DLA)  
(Smoluchosky 1917)

$k_s$  constant for slow aggregation (RLA)  
(Verwey et Overbeek 1948)

DLVO:  $W \approx \frac{1}{\kappa_D 2a} \exp\left(\frac{V_{max}}{k_B T}\right)$

Arrhenius like equation

## Slow or rapid aggregation ?

Half life time

$$t_{1/2} = \frac{1}{k_s n_0} = \frac{W}{8\pi D a n_0}$$

*Characteristic time for the aggregation*

$$t_{1/2} = \frac{W}{6D\phi} a^2$$

AN : Half life times for 200 nm particles at a volume fraction of  $10^{-4}$

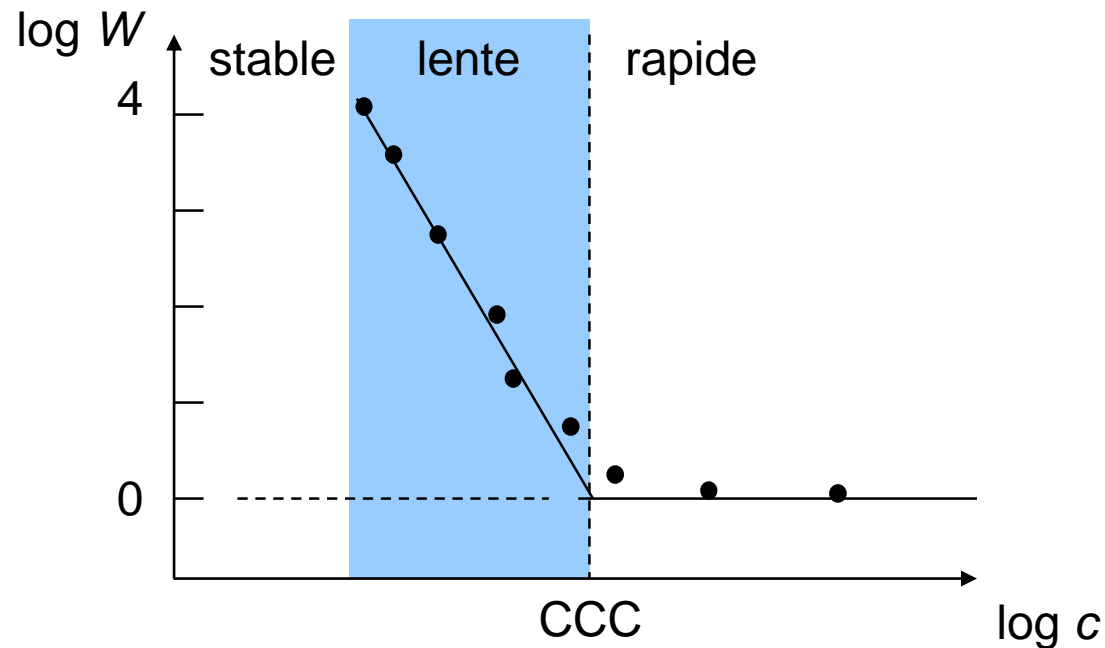
I (M)	$10^{-3}$	$10^{-2}$	0.05
$K_d$ (nm)	9.7	3.1	1.4
$V_{\max}$	23	10	0
W	$5 \cdot 10^8$	300	1
$t_{1/2}$ (s)	$4 \cdot 10^9$	$2.6 \cdot 10^3$	7.6
	1.1 siècle	43 min	7.6 s
	metastable	slow	rapid

## Slow or rapid aggregation

For a coagulant at a concentration  $c$  (mol/dm<sup>3</sup>) :

$$\log_{10} W \approx K_1 \log_{10} c + K_0$$

$$K_1 \approx -2 \times 10^9 \frac{\Upsilon_0^2 a}{z^2}$$

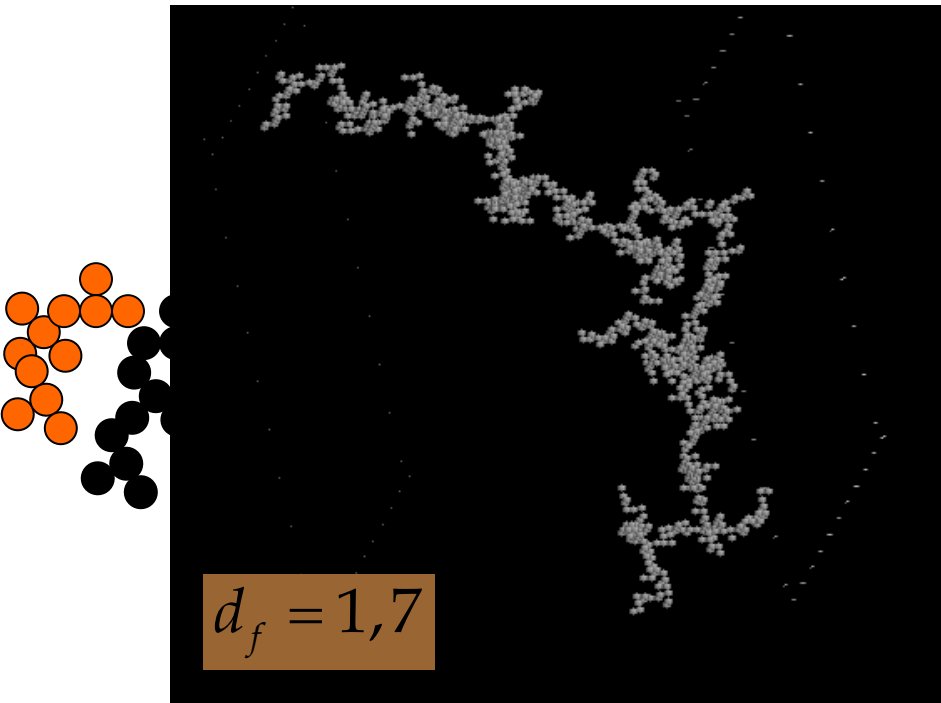


Slow coagulation when  $\log W < 4$  ( $W < 10^4$ ) :  $V_{\max} \sim 15k_B T$

Stability conditions :  $V_{\max} > 15k_B T$

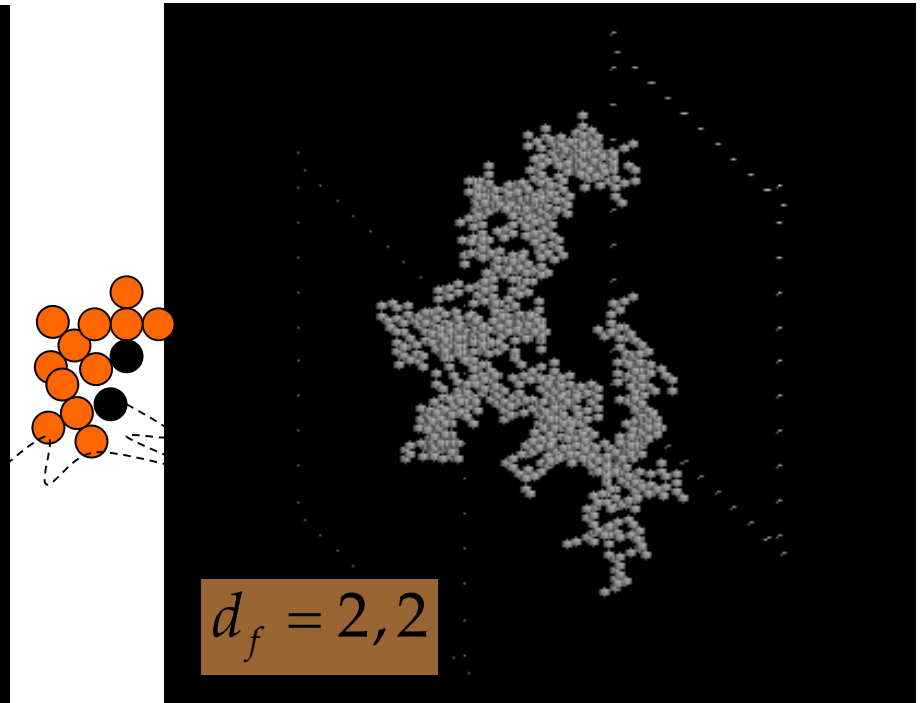


## Perikinetic aggregation and fractal dimension



rapid aggregation =  
limited by the diffusion

$$N_{agg} = \left( \frac{R_{agg}}{a} \right)^{d_f}$$



slow aggregation :  
limited by interactions

$$\phi_{agg} = \left( \frac{R_{agg}}{a} \right)^{d_f - 3}$$

$$\rho_{agg} = \rho_s \phi_{agg} + (1 - \phi_{agg}) \rho$$

## Exercise

Particles of 100 nm in size (diameter) are aggregated. The size of aggregates is 10  $\mu\text{m}$  and the fractal dimension is 2.3.

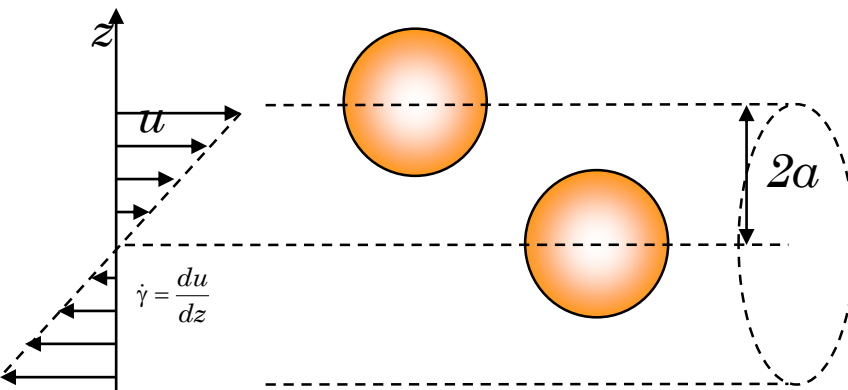
Conclude on the aggregation regime.

Compute the density of aggregate and the number of primary particles in an aggregate.

Compare the settling velocities for primary particles and aggregates. Discuss the assumptions made to compute the settling velocities.

Données : *masse volumique des particules primaires : 1500 kg/m<sup>3</sup>*

Orthokinetic aggregation



Orthokinetic constant

$$k_o = \frac{16}{3} \dot{\gamma} a^3$$

For primary particles :

$$\frac{dn}{dt} = -\frac{4\dot{\gamma}\phi}{\pi} n$$

$$\frac{n}{n_0} = \exp\left(-\frac{4\dot{\gamma}\phi}{\pi} t\right)$$

For a given volume fraction, the aggregate growing kinetics is a function of

$\dot{\gamma}t$  « Camp number"

Thomas et Camp (1953)

For a turbulent mixing

Camp & Stein, 1943

$$\langle \dot{\gamma} \rangle = \sqrt{\rho \frac{\varepsilon}{\mu}}$$

mixing power per unit of fluid mass

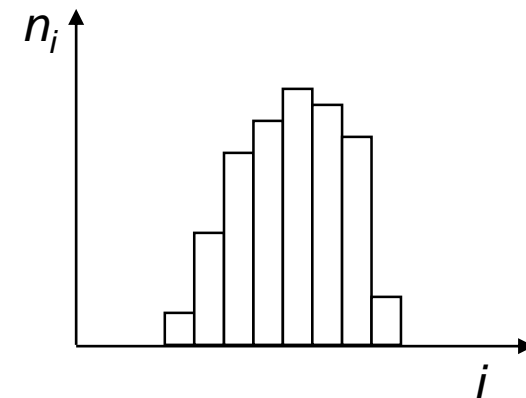
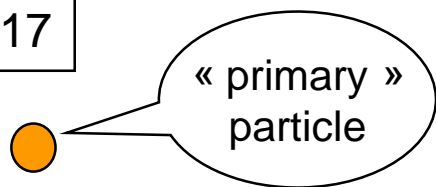


# Aggregation

## Aggregation and population balances

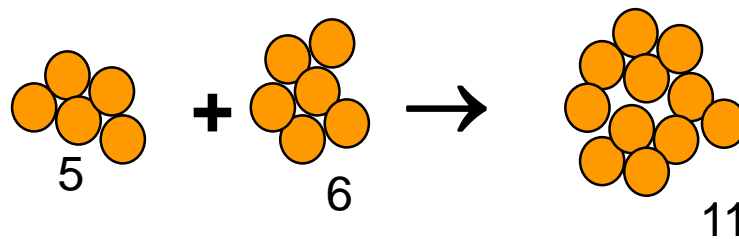
But: the size distribution changes with time  
(previous approaches are valid only for the first aggregation time)

Smoluchowski 1917



$n_i$  = number of particles containing  $i$  primary particles

Aggregation of aggregates have to be accounted !



constante de cinétique  
de 2<sup>e</sup> ordre

Number of collisions between particles of population  $i$  and particles of population  $j$  ( $\text{m}^{-3} \text{s}^{-1}$ )

$$\dot{n}_{ij} = k_{ij} n_i n_j$$



## Population balance equations

$$\frac{dn_k}{dt} = \frac{1}{2} \sum_{\substack{i+j=k \\ i=1}}^{i=k-1} k_{ij} n_i n_j - n_k \sum_{k=1}^{\infty} k_{ik} n_i$$

Birth

Death

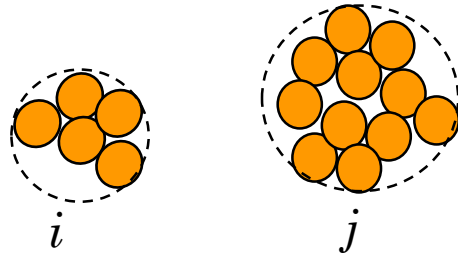
$$\begin{aligned} \frac{dn_1}{dt} &= -k_{11} n_1^2 - k_{12} n_1 n_2 - k_{13} n_1 n_3 \dots \\ \frac{dn_2}{dt} &= \frac{1}{2} k_{11} n_1^2 - k_{12} n_1 n_2 - k_{23} n_2 n_3 \dots \\ \frac{dn_3}{dt} &= \frac{1}{2} k_{12} n_1 n_2 - k_{13} n_1 n_3 - k_{23} n_2 n_3 \dots \end{aligned}$$

Total particles concentration :

$$\frac{dn}{dt} = -k_a n^2 \quad \text{if } k_{ij} = k_{11} = 2k_a$$

$$n = \frac{n_0}{1 + k_a n_0 t} \quad \text{at } t = 0: n = n_1 = n_0$$

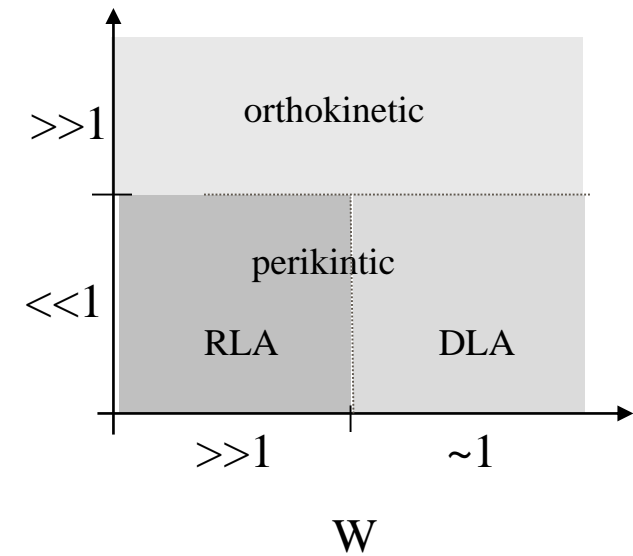
## Estimation of aggregation modes



$$Pe = \frac{F_{hydr}}{F_{br}} = \frac{6\pi\mu_w \dot{\gamma} a_i^3}{kT}$$

$$= 4.6 \cdot 10^{18} \dot{\gamma} a_i^3$$

Pe



- Brownian diffusion (perikintic aggregation),  $Pe \ll 1$

$$k_{ij} = \frac{1}{W} \frac{2k_B T}{3\mu} \frac{(a_i + a_j)^2}{a_i a_j}$$

- Velocity gradient (orthokinetic aggregation),  $Pe \gg 1$

$$k_{ij} = \frac{4}{3} \dot{\gamma} (a_i + a_j)^3$$

- Differential settling

$$k_{ij} = \left( \frac{2\pi g}{9\mu_w} \right) (\rho_s - \rho) (a_i + a_j)^3 (a_i - a_j) \quad 42$$



# Aggregation

## Application : coagulation/flocculation of waste water (1)

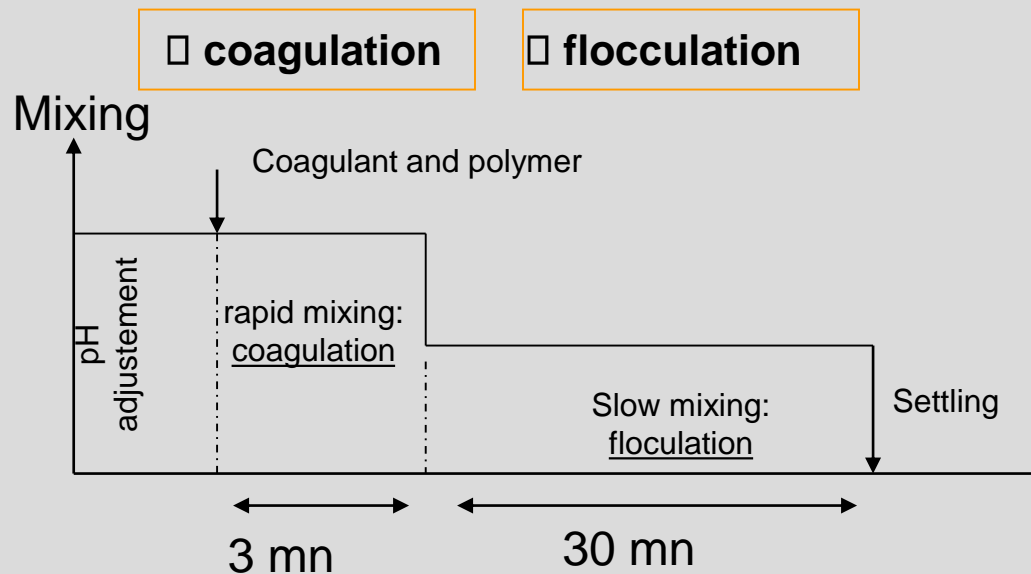
To treat water, the aggregation process occurs in two steps

### □ coagulation

destabilisation of colloidal particles -> aggregate by adding in a rapid mixing zone a coagulant (salt with high valency) to screen charges

### □ flocculation

reversible formation of flocs between coagulated particles in a slow mixing zone by inter-particle bonding with polymer

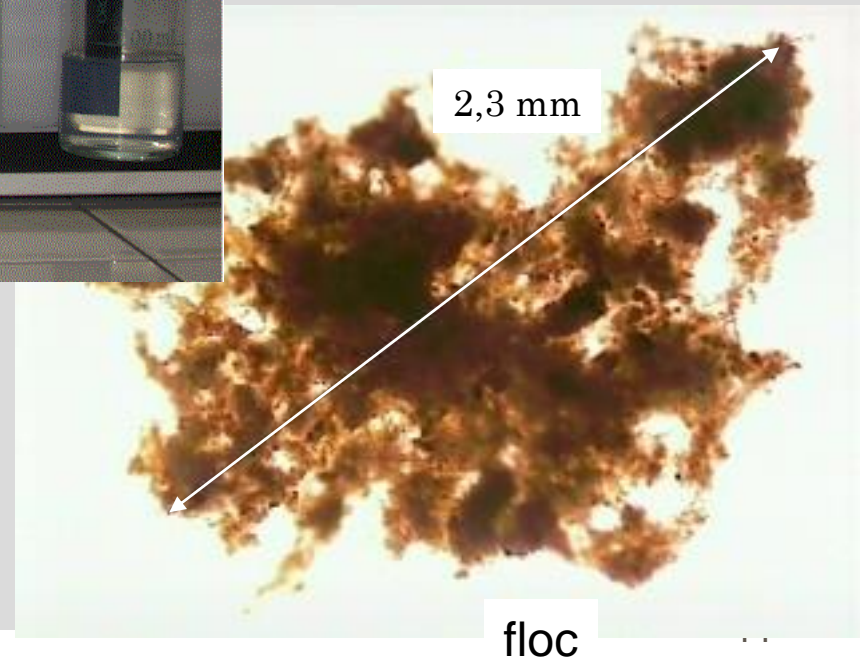
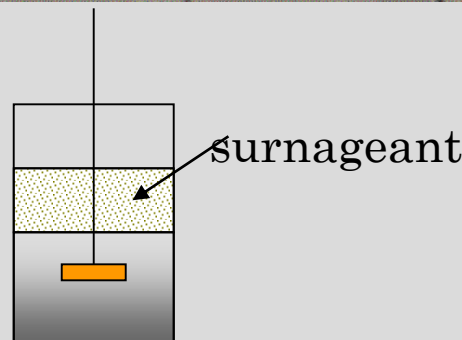
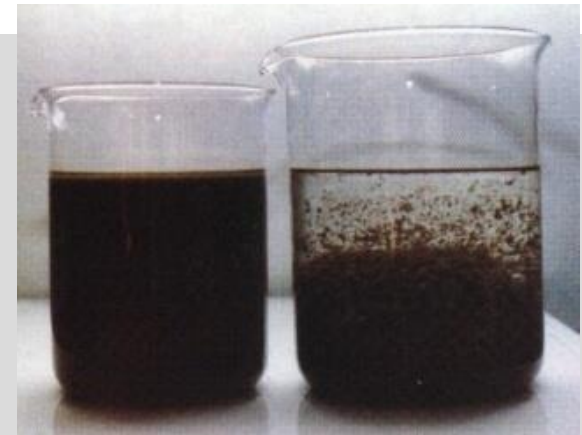




# Aggregation

## Application : coagulation/floculation in the lab

### Jar test



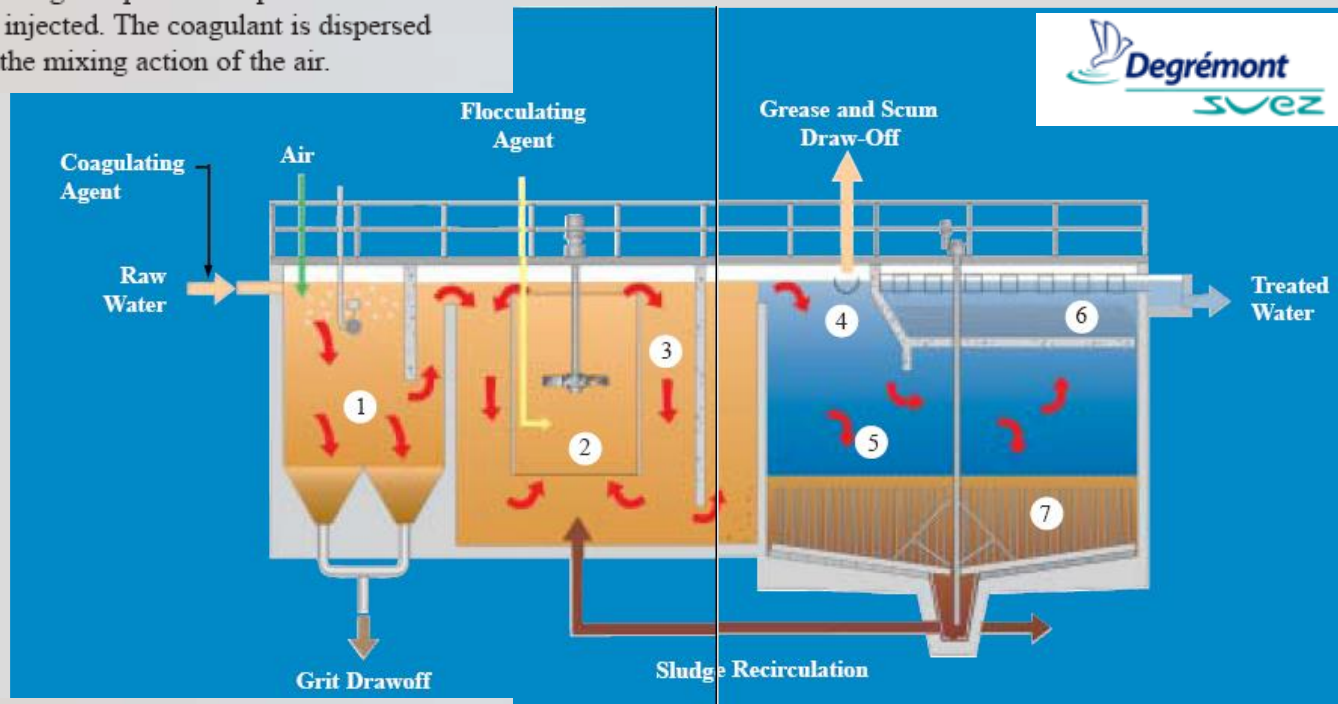


## Application : coagulation/flocculation at industrial level

### Operating Diagram

**Zone 1: Grit removal/coagulation** The raw water enters an air-mixing zone where grit separation is performed and a coagulating agent is injected. The coagulant is dispersed in the storm water by the mixing action of the air.

### A l'échelle industrielle : Densadeg Degrémont



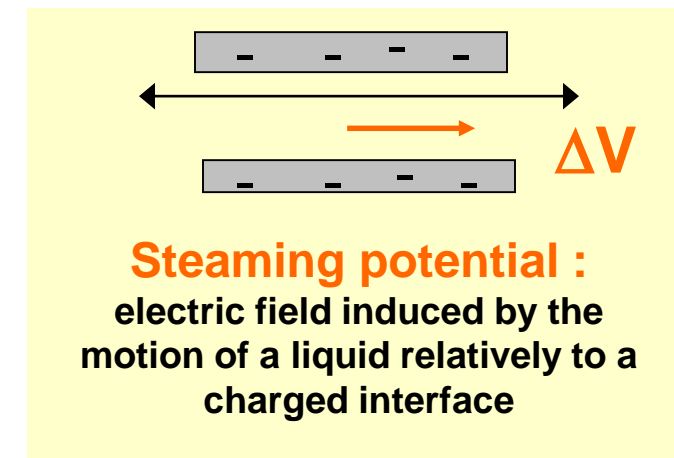
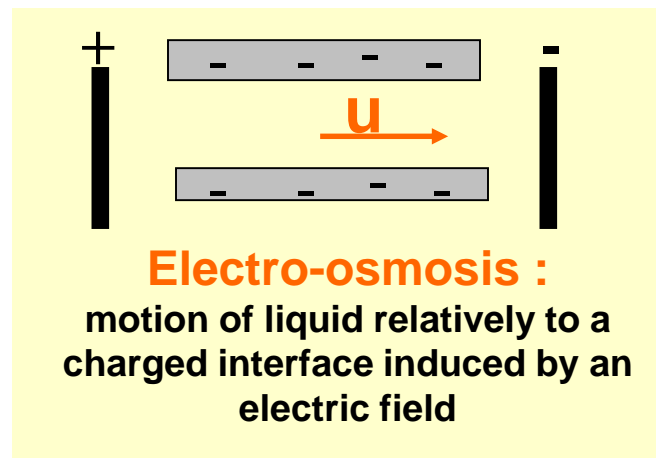
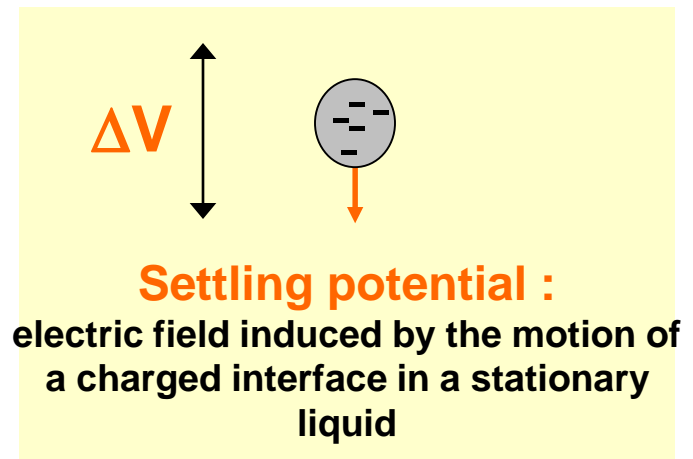
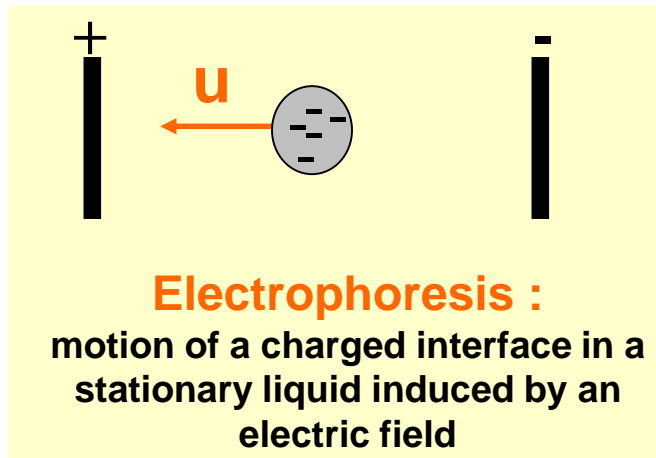
#### Zone 2: Flocculation, first stage

The water then flows into a second zone for intense internal recirculation and mixing by an axial-flow turbine. Here, a flocculating agent is added, together with thickened sludge recirculated through an external system. The recirculated sludge accelerates the flocculation process and ensures the formation of dense floc particles of homogeneous size.

#### Zone 3: Flocculation, second stage

The transition to the settling stage is accomplished in this zone. The process is a plug-flow reactor where the flocculation process continues and the grease and scum start to separate out.

**When the motion of an electrostatic double layer and an electric field interplay ...**



## Electrophoresis :

$\lambda_D \gg a$  Assumption of a ponctual charge

**Force balance :**

$$u = \frac{qE}{6\pi\mu a}$$

**With:**  $\zeta = \frac{q}{4\pi\epsilon a}$

$$u = \frac{2\epsilon\zeta E}{3\mu}$$

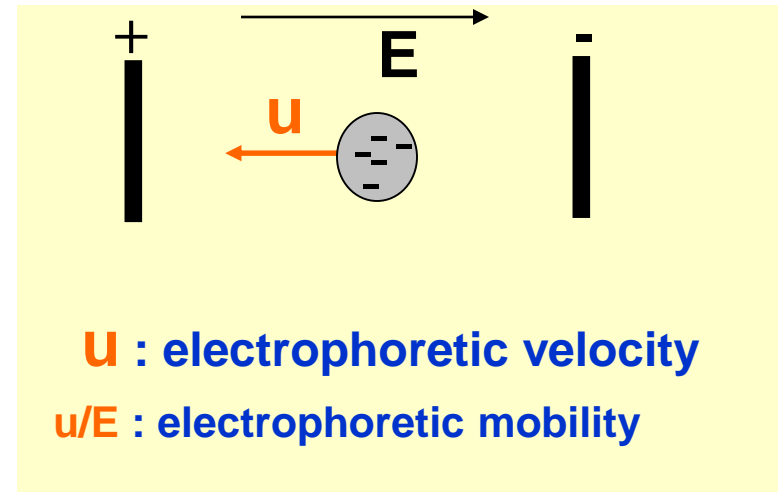
**Hückel equation**

$\lambda_D \ll a$  Assumption of a plane double layer

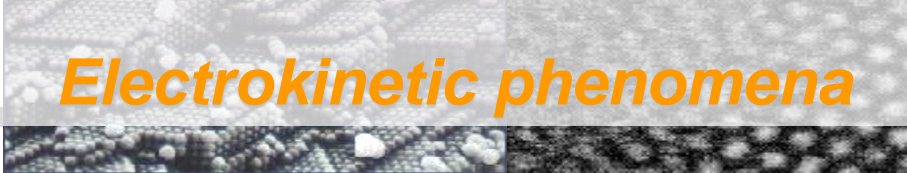
**Momentum transport equation :**

$$u = \frac{\epsilon\zeta E}{\mu}$$

**Helmholtz-Smoluchowski equation**

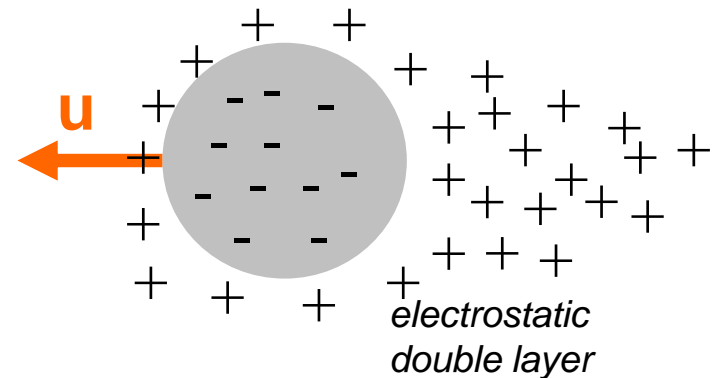


**Applications :** separation,  
particle and macromolecule  
charge analysis



finite  $\lambda_D$   $\rightarrow$  Electrophoretic retardation

The counter-ions of the double-layer move in the opposite direction (electro-osmosis) and slow down the electrophoresis



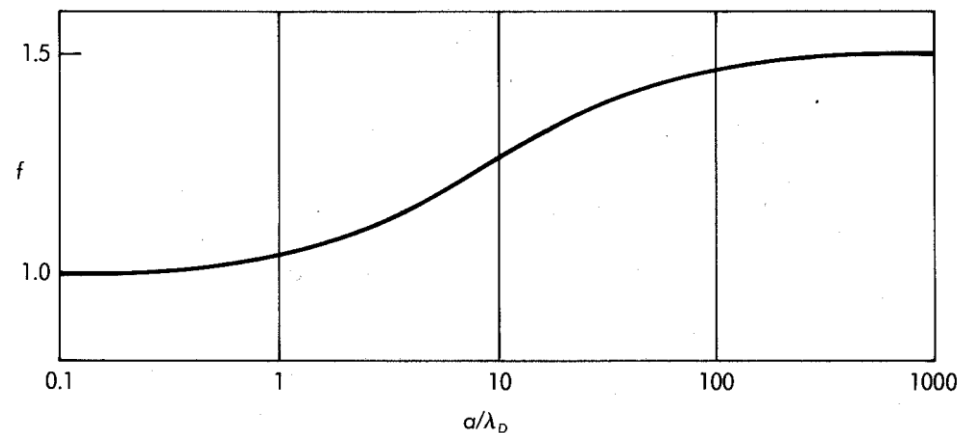
$$u = \frac{2\varepsilon\zeta E}{3\mu} f\left(\frac{a}{\lambda_D}\right)$$

Henry equation function (1931)

$\rightarrow \frac{a}{\lambda_D} \rightarrow 0 \quad f\left(\frac{a}{\lambda_D}\right) \rightarrow 1$   
 (small particle, dilute solution)

$\rightarrow \frac{a}{\lambda_D} \rightarrow \infty (> 100) \quad f\left(\frac{a}{\lambda_D}\right) \rightarrow 1,5$

$$f(x) \approx 1 + 0,5 / \left[ 1 + \left\{ \frac{5}{2x} (1 + 2e^{-x}) \right\}^3 \right]$$



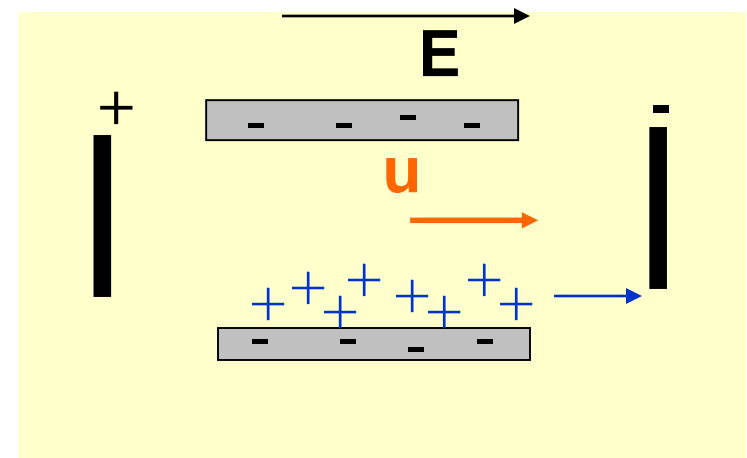
Ohshima 1994



## Electro-osmosis (discovered by F.F. Reuss 1809) :

Under an electric field, the counter-ions (in the double layer near the surface) migrate and due to viscous drag the water is drawn by the ions and flows.

**u** : electro-osmotic velocity



if the porous radius  $\gg 1/\kappa$   $f(\kappa a) \rightarrow 1,5$

$$u = -\frac{\varepsilon \zeta E}{\mu}$$

AN :  $\zeta = 100 \text{ mV}$   
 $E = 1000 \text{ V.m}^{-1}$   
 $u = 10^{-4} \text{ m.s}^{-1}$

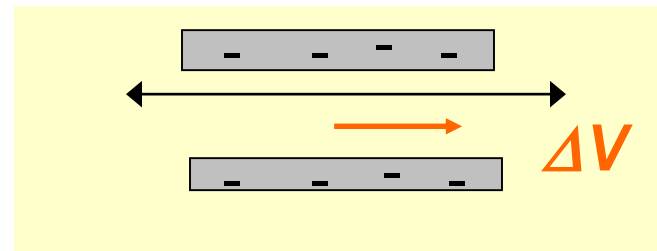
**Applications** : dewatering,  
 transfer in biological membrane

### Streaming potentiel :

Charge transport  $\rightarrow$  Streaming current  $\rightarrow$  potential difference

$$\Delta V = \frac{\varepsilon \zeta}{\mu k} \Delta p$$

$k$  conductivity of the bulk solution



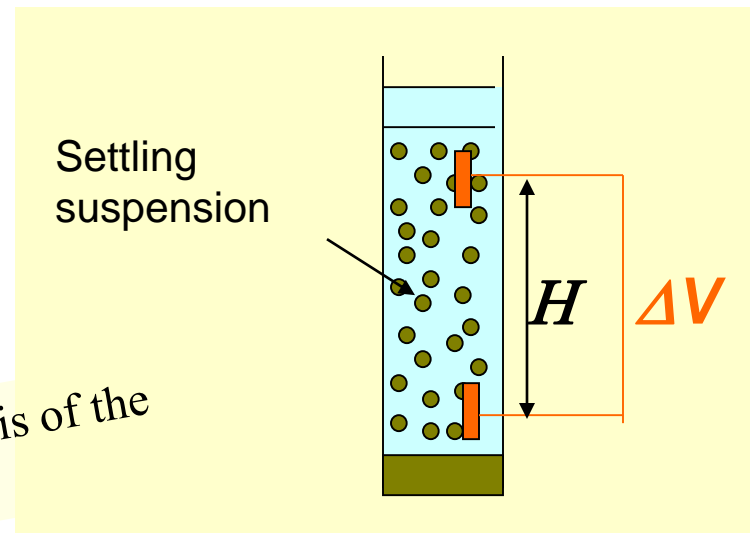
**Applications :** analysis of surface charge

### Settling potential :

$$\frac{\Delta V}{H} = - \frac{6\pi n U a \varepsilon \zeta}{k}$$

$U$  : particle fall speed

$n$  : particles per unit volume



**Applications :** analysis of the particle charge ?

## Applications (1)

### Microelectrophoresis

Measurement of the motion of charged particles or macromolecules under microscopy (when size is  $> 1 \mu\text{m}$ ) or by laser interferometer

velocity  $\rightarrow$  **zéta potential**

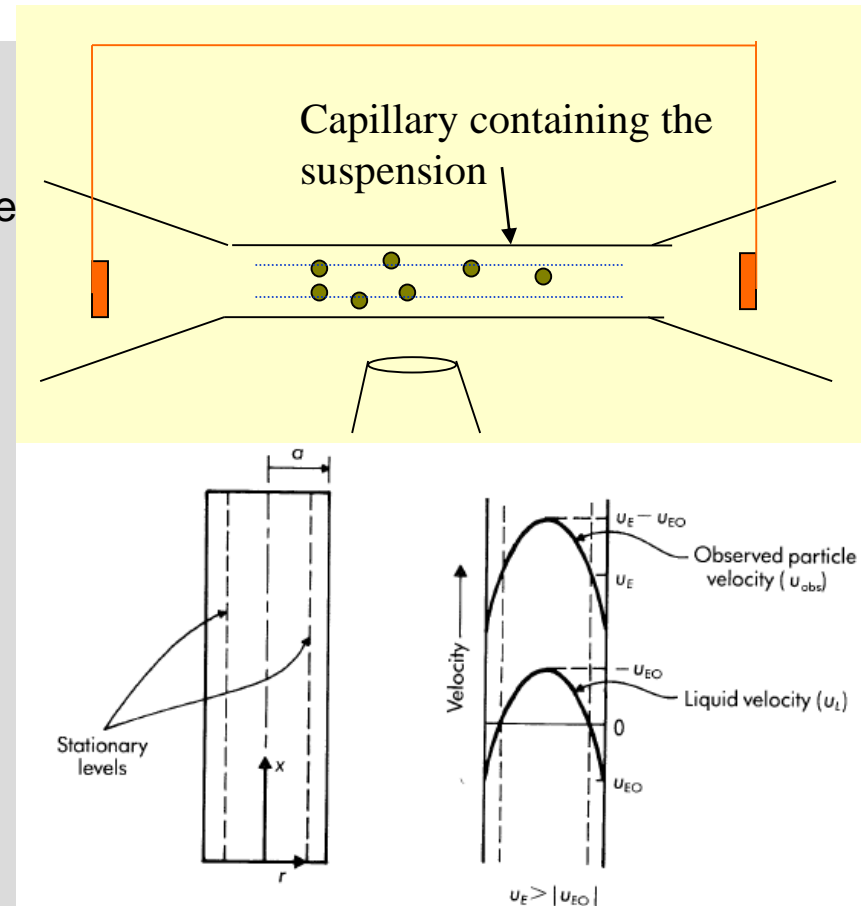
#### Problem :

The charge of the capillary leads to an electro-osmosis phenomena that modify the motion of particles

#### Solution :

Measurement of the electrophoretic velocity at stationary plane

$$u_L = u_{EO} \left( 2 \frac{r^2}{a^2} - 1 \right) \quad \rightarrow \quad \frac{r_{stat}}{a} = \frac{1}{\sqrt{2}}$$



Physicochemical hydrodynamics : An introduction, Wiley Inter Science, R. F. Probstein (1994)

No slipping conditions are still existing in microfluidic?

E. Lauga et al. *Springer Handbook of Experimental Fluid Mechanics* (2007)

## Applications (2)

### zone electrophoresis

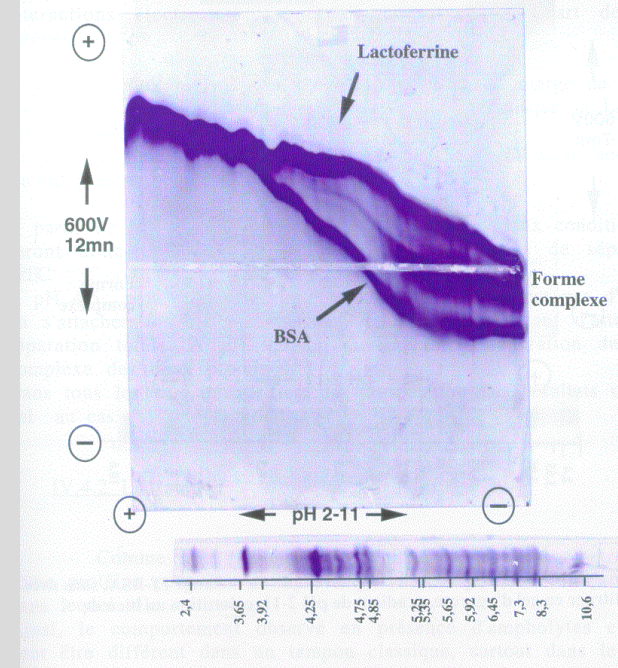
Migration of charged solutes in a gel

+ *reduction of convection (due to Joule effect)*

- *electroosmosis and capilarity make the estimation of the solute charge impossible*

### Use for qualitative analysis

*Electrotitring curve in an agarose gel with a pH gradient*



## L'ÉLECTROPHORÈSE SÉPARE LE BON DU MAUVAIS

**Q**u'elle soit endogène ou exogène, l'EPO se présente sous plusieurs « isoformes ». Celles-ci diffèrent légèrement les unes des autres au niveau de groupements chimiques qui se fixent sur la molécule après sa « transduction », c'est-à-dire sa fabrication dans la cellule

suivant les instructions du code génétique. Or l'EPO exogène recombinante, fabriquée par des cellules animales génétiquement modifiées, n'a pas le même « spectre » d'isoformes que la molécule 100 % humaine, et on peut le voir par séparation électrophorétique.

La méthode consiste à faire migrer les molécules à l'intérieur d'un gel déposé sur une plaque, par application d'un champ électrique. Les isoformes n'ayant pas la même charge électrique migrent à des vitesses différentes et se séparent. Le résultat se présente,

après révélation de la plaque, sous forme d'un spectre de taches. Or le spectre de l'EPO recombinante diffère nettement de celui de la molécule endogène. Le Laboratoire national de dépistage du dopage a testé sa méthode sur 102 échantillons urinaires préle-

vés lors du Tour de France 1998 et conservés par congélation. Sur 28 échantillons, dont une analyse préalable avait révélé une teneur anormalement élevée d'EPO, 14 ont été soumis au test par électrophorèse et ont révélé la présence d'EPO exogène.

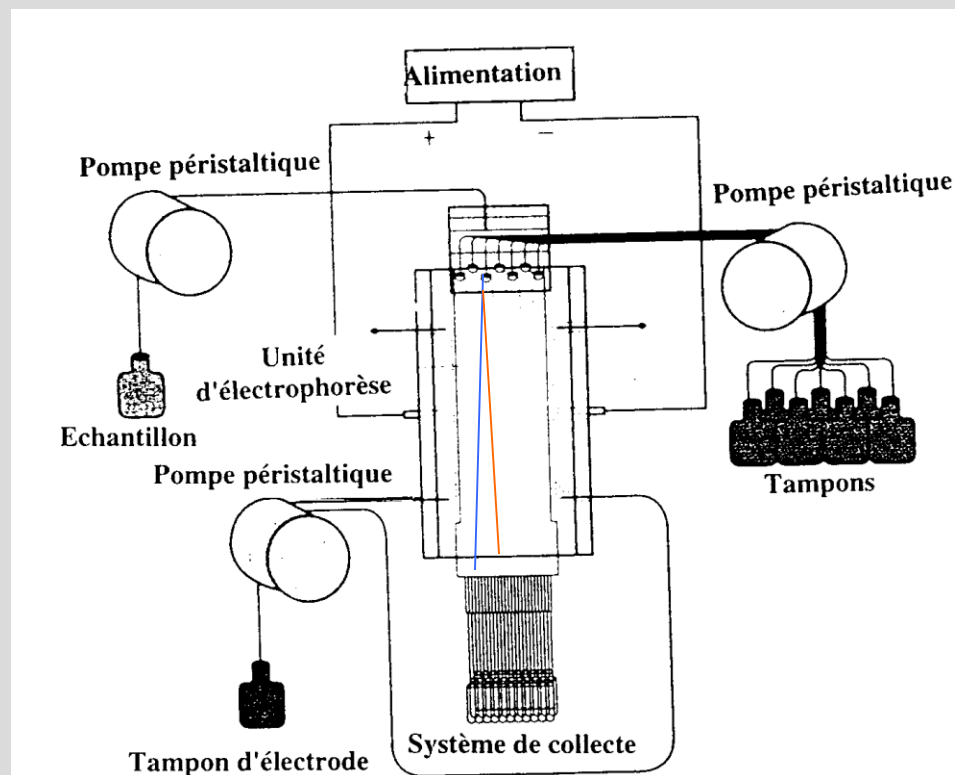
## Applications (3)

### Continuous electrophoresis

Electrophoresis during the liquid streaming

- + *important separation*
- *Joule effect*

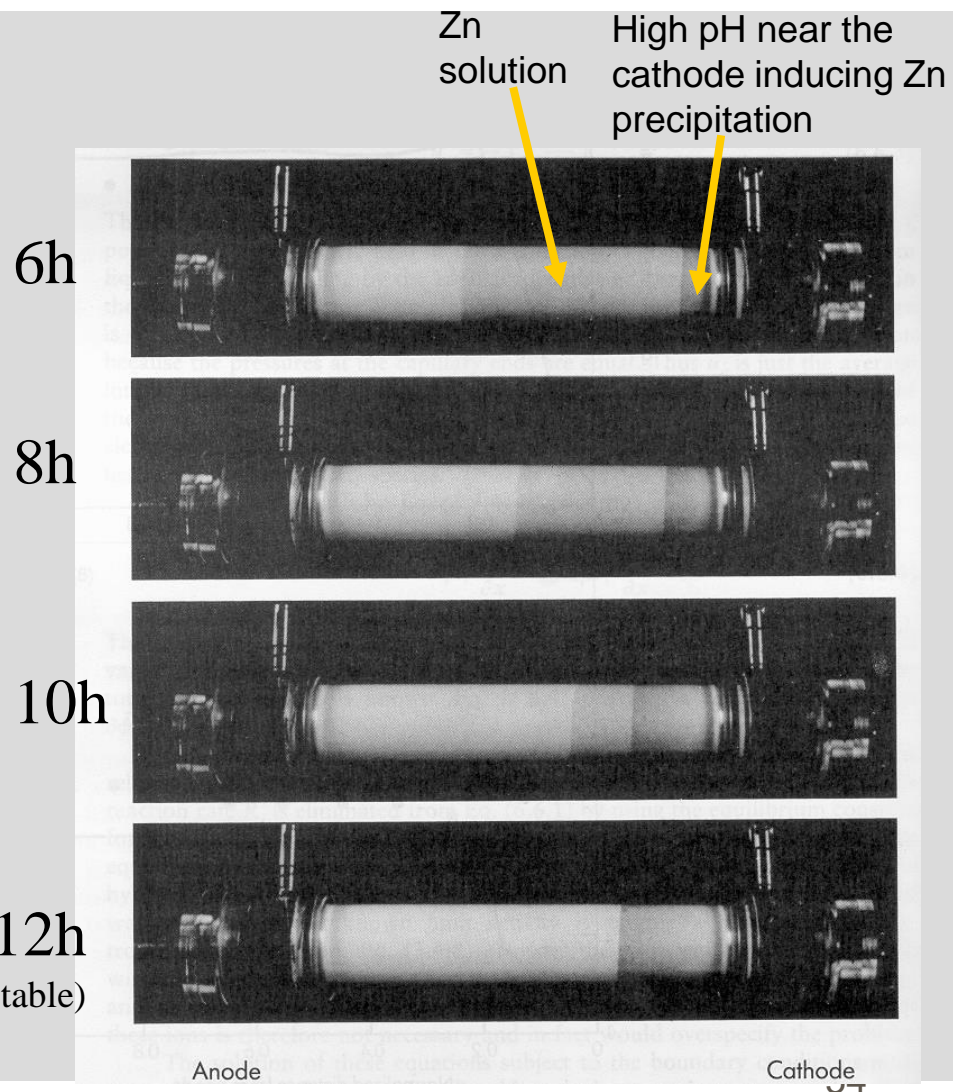
**Use for the purification of biochemical product**



## Applications (4)

### Electroosmosis for the removal of contaminants from soils

*Zinc ( $8 \text{ mol/m}^3$ ) removal from a cylindrical clay sample 0.2 m long across which 8 V is applied.*



## Osmotic pressure and $a_w$

Chemical potentiel :

$$\mu_i = \mu_i^0 + V_i P + RT \ln a_i$$

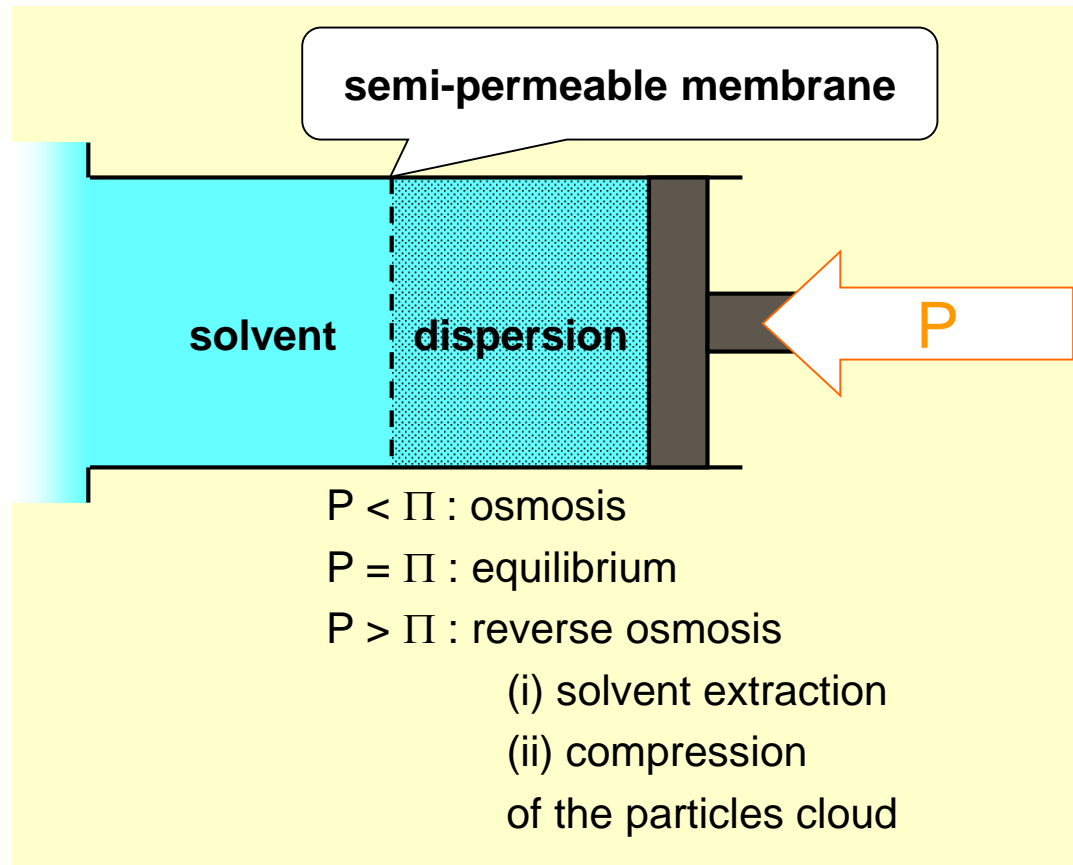
At equilibrium :

$$\mu_w^s = \mu_w^d$$

$$V_w P_0 = V_w (P_0 + \Pi) + RT \ln a_w$$

$$V_w \Pi = -RT \ln a_w$$

water activity : represent the water availability "free water" and then the interactions between particles

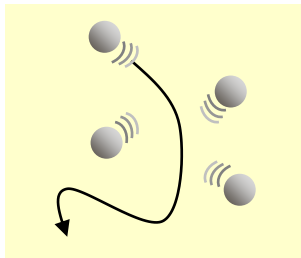


Repulsion between particles	↗	$a_w$	↘	$\Pi$	↗
Attraction between particles	↗	$a_w$	↗	$\Pi$	↘
Attraction solvent-particles	↗	Repulsion	↗	$a_w$	↘
				$\Pi$	↗



## Osmotic pressure and interactions

Gas and ideal solution:  $\ln a_w \approx \ln x_w = \ln(1 - x_p) \approx -x_p - \frac{x_p^2}{2} - \dots$

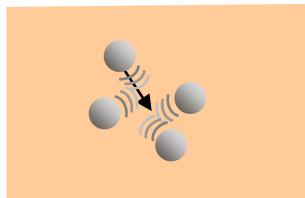


$$\Pi = \frac{x_p}{V_w} RT \approx CRT = nk_B T$$

$\downarrow$  mol/m<sup>3</sup>       $\downarrow$  m<sup>-3</sup>

van't Hoff law

Non-ideal solution and dispersion of interacting particles or macromolecules :



$$\Pi = nk_B T - \frac{2\pi}{3} n^2 \int_0^\infty r^3 g(r) \frac{dV_T}{dr} dr$$

*Theoretical relationship*

colloidal interactions (DLVO)

Averaged molecular mass

$$\frac{\Pi \langle M \rangle_n}{cRT} = 1 + \Gamma_2 c + \Gamma_3 c^2 + \dots$$

*Semi-empirical relationship for macromolecules*

Mass concentration in g/m<sup>3</sup>

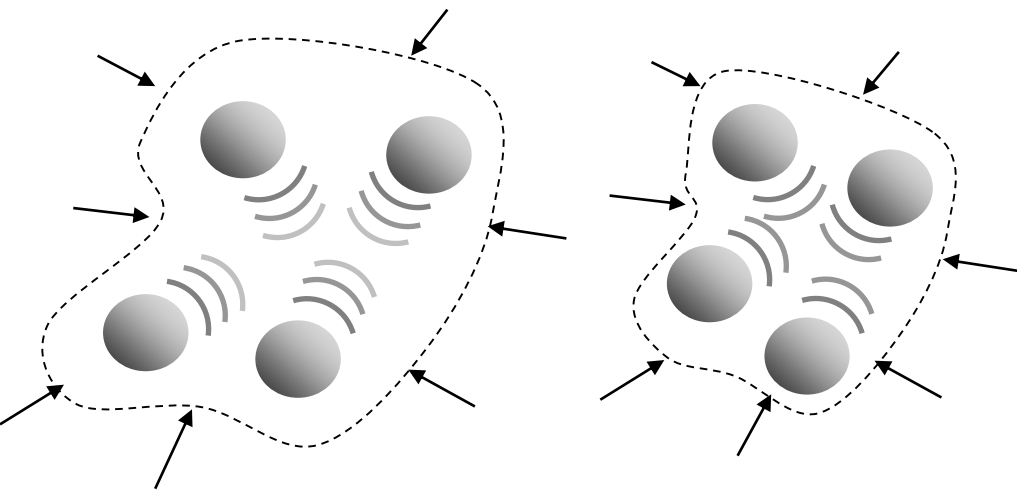
Virial coefficient

function of colloidal interactions, hydration ...



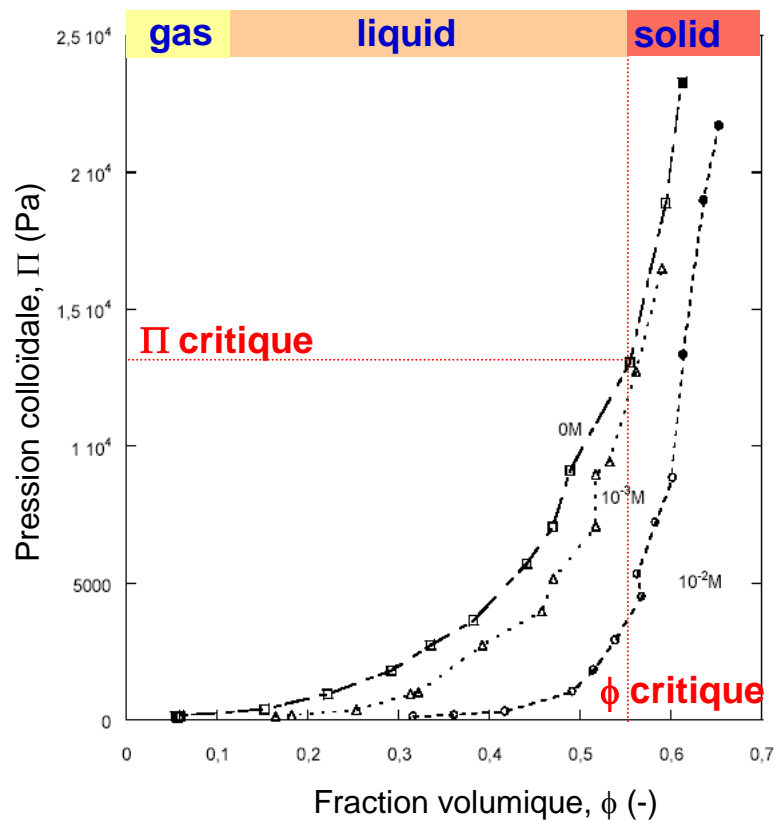


### Experimental highlights



The osmotic pressure is dependent of the ionic strenght

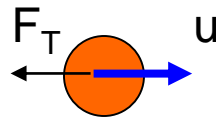
$I$  ↗  
 repulsion ↘  
 $\Pi$  ↘  
 compression resistance ↘  
 $\Phi$  ↗ (for an applied pressure)





## Mobility, $m$ , of a dispersion

Diluted suspension of spheres

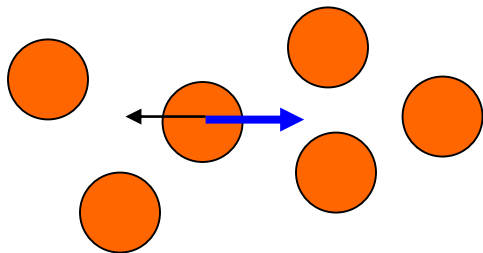


Stokes law

$$F_T = 6\pi\mu a u$$

$$m = \frac{1}{6\pi\mu a}$$

Concentrated suspension of spheres

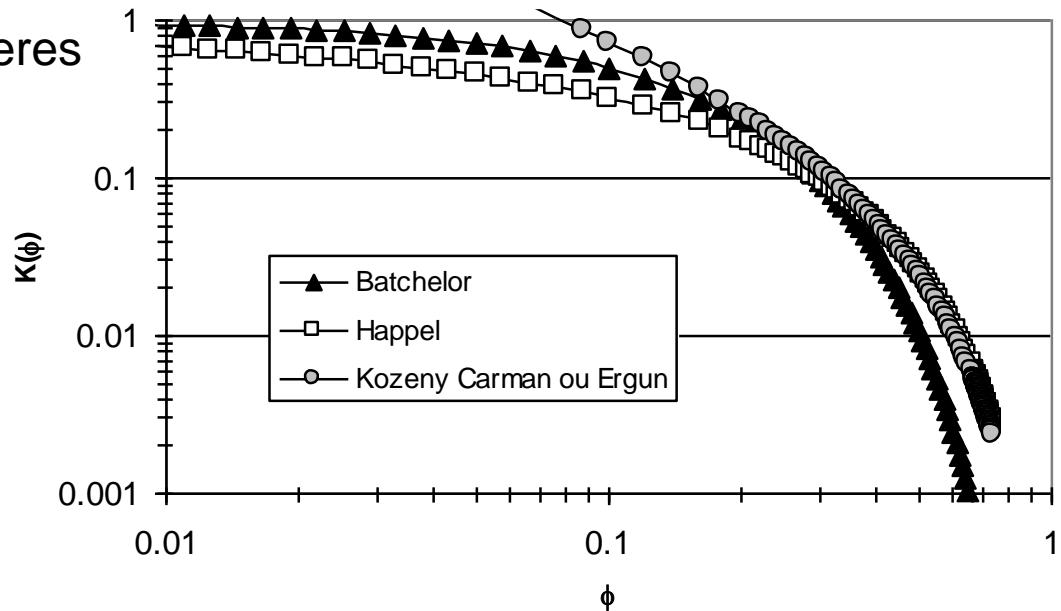


$$F_T = \frac{6\pi\mu a u}{K(\phi)}$$

$$m(\phi) = \frac{K(\phi)}{6\pi\mu a}$$

hydrodynamique  
coefficient  
(settling coefficient)

$$K(\phi) = \frac{u_{sed}(\phi)}{u_{sed_0}}$$



Happel function

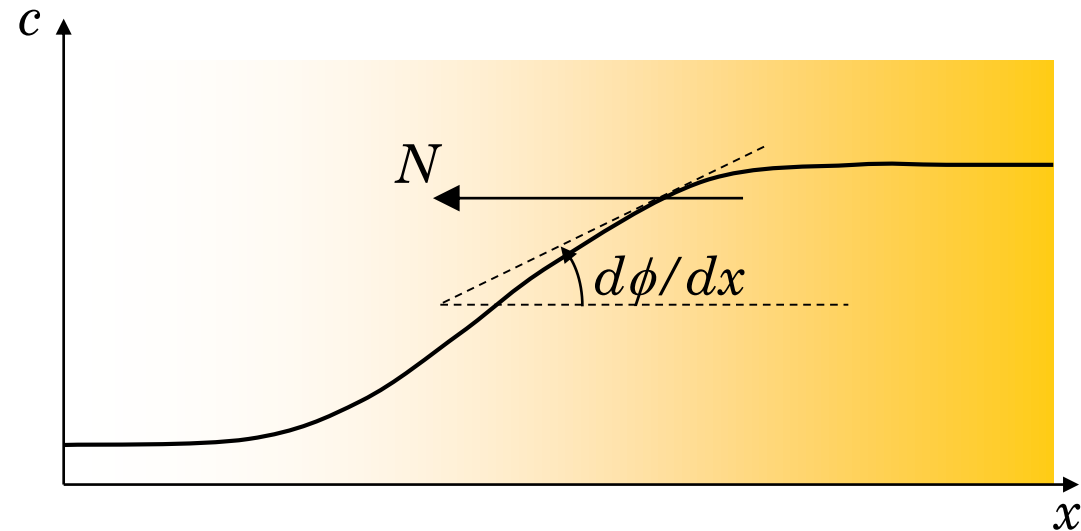
$$H(\phi) = \frac{1}{K(\phi)} = \frac{6 + 4\phi^{\frac{5}{3}}}{6 - 9\phi^{\frac{1}{3}} + 9\phi^{\frac{5}{3}} - 6\phi^2}$$

## Fickian diffusion

Transport of mass  
from concentrated to diluted zones :

**Mission :**  
Back to equilibrium

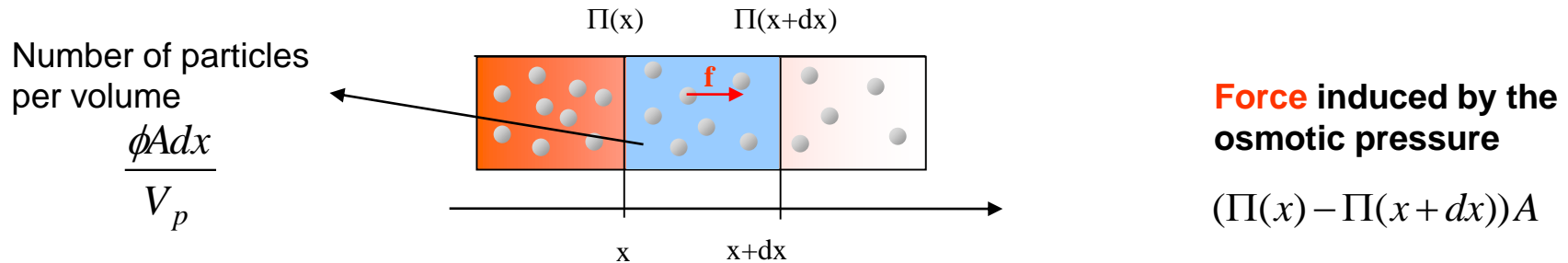
**Means :**  
Brownian motion



$D$  : diffusion coefficient

$$N = -D \frac{d\phi}{dx}$$

The key : a link between diffusion, mobility and osmotic pressure



Force per particle  $f = -\frac{V_p}{\phi} \frac{\partial \Pi}{\partial x} = -\frac{V_p}{\phi} \left( \frac{\partial \Pi}{\partial \phi} \right) \left( \frac{\partial \phi}{\partial x} \right)$  inducing a x velocity  $u = m(\phi) f$

Mass flux  $N = u\phi = -m(\phi)V_p \left( \frac{\partial \Pi}{\partial \phi} \right) \left( \frac{\partial \phi}{\partial x} \right)$

Generalised Stokes Einstein law

Collective diffusion  
(in a gradient)

$$D = \frac{K(\phi)}{6\pi\mu a} V_p \frac{d\Pi}{d\phi}$$

$$D \xrightarrow{\phi \rightarrow 0} \frac{kT}{6\pi\mu a}$$

hydrodynamic

&

colloidal

interactions colloïdales

## Application to the filtration (1)

$$J = -\frac{k_p}{\mu} \frac{dP}{dx}$$

Permeability coefficient (m<sup>2</sup>)

Darcy law

Integration over the thickness, e, of the membrane

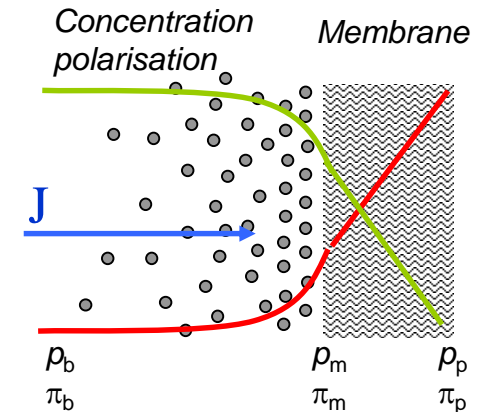
$$J = \frac{k_p}{\mu} \frac{\Delta P}{e} = \frac{L_p}{\mu} \Delta P = \frac{\Delta P}{\mu R_m}$$

Permeability (m)

Hydraulic resistance (m<sup>-1</sup>)

with accumulation  
at the membrane

$$J = \frac{\Delta P - (\Pi_m - \Pi_p)}{\mu R_m}$$



— Osmotic pressure  $\pi$

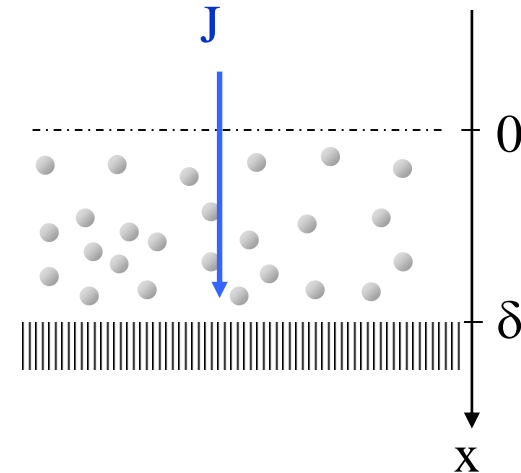
— Liquid static pressure  $p$

## Application to the filtration (2)

In permanent regime:  
mass transport by convection is counterbalanced by diffusive transport

Effect of interfacial  
phenomena

$$N = J\phi - \frac{K(\phi)}{6\pi\mu a} V_p \frac{d\Pi}{dx} = 0$$



In cross flow filtration (accumulation in a boundary layer  $\delta$ )

Without interactions

$$\frac{Jdx}{D_0} = \frac{d\phi}{\phi} \quad Pe = \frac{J\delta}{D_0} = \ln\left(\frac{\phi_m}{\phi_b}\right)$$

With interactions

$$Pe = \frac{V_p}{kT} \int_{\Pi_b}^{\Pi_m} \frac{K(\phi)}{\phi} d\Pi$$

## Application to the settling (1)

Gravity

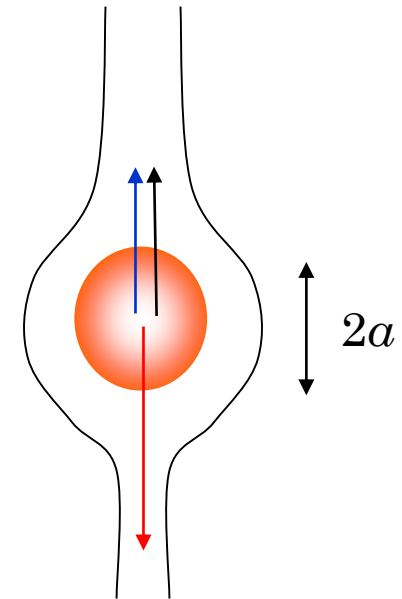
Archimeda

drag

$$V_p (\rho_p - \rho_w) g = \frac{6\pi\mu a}{K(\phi)} u$$

For diluted suspension of spheres:

$$u = \frac{2}{9} \frac{a^2 (\rho_p - \rho_w) g}{\mu}$$

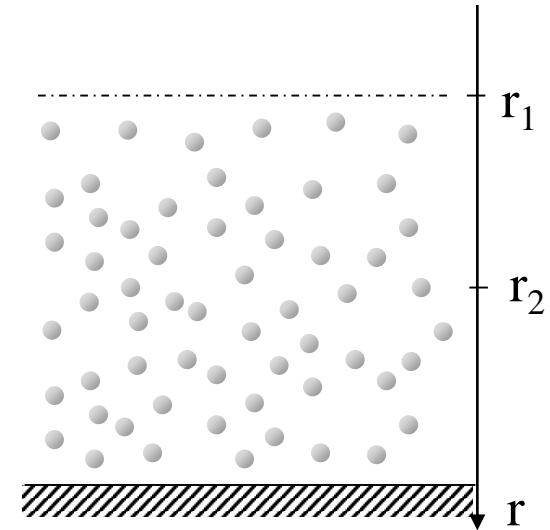


## Application to the settling (2)

$$N = u_{sed}\phi - D(\phi)\frac{d\phi}{dx} = 0$$

Effect of interfacial  
phenomena

$$(\rho_p - \rho_w)g\phi - \frac{d\Pi}{dr} = 0$$



Without interactions

$$\ln\left(\frac{\phi_2}{\phi_1}\right) = \frac{V_p(\rho_p - \rho_w)g}{kT}(r_2 - r_1)$$

With interactions

$$(\rho_p - \rho_w)g(r_2 - r_1) = \int_{\Pi_1}^{\Pi_2} \frac{d\Pi}{\phi}$$



## Application to the centrifugation (2)

Effect of interfacial  
phenomena

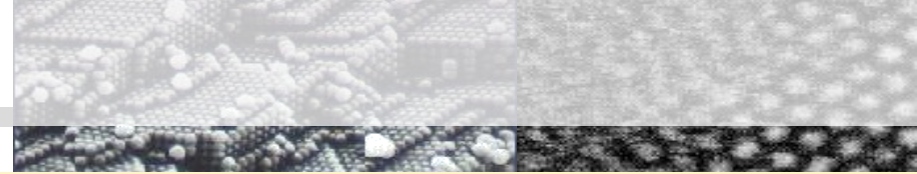
$$(\rho_p - \rho_w)\omega^2 r\phi - \frac{d\Pi}{dr} = 0$$

Without interactions

$$\ln\left(\frac{\phi_2}{\phi_1}\right) = \frac{V_p(\rho_p - \rho_w)}{kT}\omega^2(r_2^2 - r_1^2)$$

With interactions

$$\frac{V_p(\rho_p - \rho_w)}{kT}\omega^2(r_2^2 - r_1^2) = \frac{V_p}{kT} \int_{\Pi_1}^{\Pi_2} \frac{d\Pi}{\phi}$$



Un récipient contient une suspension à 1 g/L de particules à une hauteur de 10 cm. A 25 °C, l'équilibre s'établit entre la diffusion et la sédimentation sous l'effet de la gravité terrestre ( $g = 9,8 \text{ m s}^{-2}$ ).

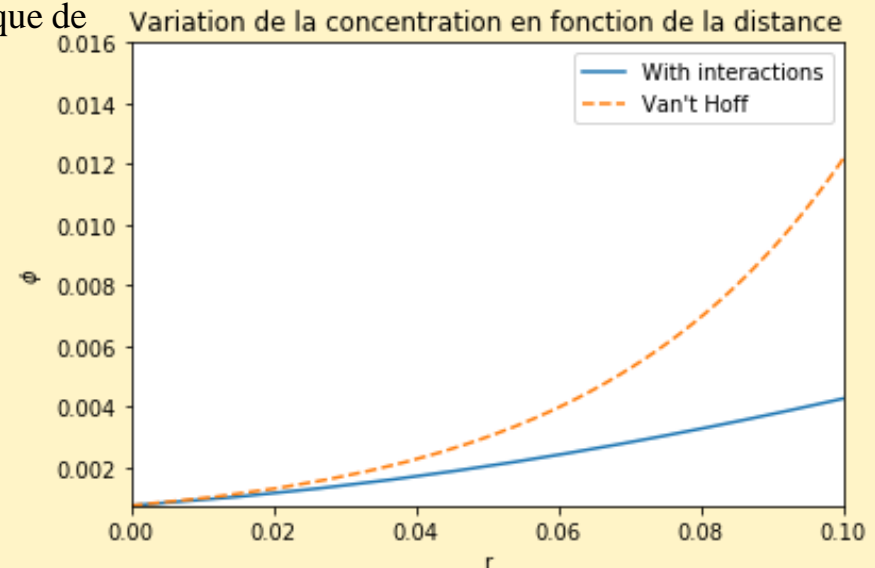
Quel est la concentration en bas du récipient, si ces particules sphériques ont un diamètre de 40 nm et une masse volumique de 1,35 g/mL en l'absence d'interactions ?

Vitesse de sédimentation :  $= 3,05 \cdot 10^{-10} \text{ m/s}$

Coefficient de diffusion :  $= 1,09 \cdot 10^{-11} \text{ m}^2/\text{s}$

$$\ln\left(\frac{\phi_2}{\phi_1}\right) = \frac{u_{sed}}{D} (r_2 - r_1)$$

$$C_2 = 16.4 \text{ g/L}$$



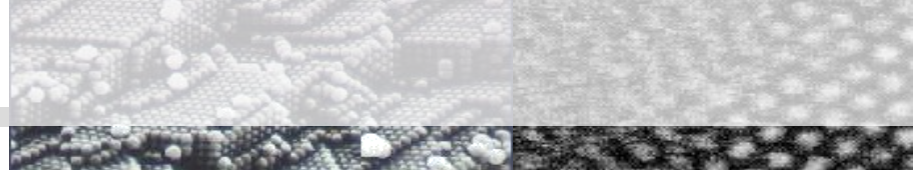
On considère maintenant, qu'en présence d'interactions répulsives entre les particules, la pression osmotique ne varie plus de façon linéaire avec la concentration mais se traduit par un second coefficient de Viriel de  $0,01 \text{ Pa}/(\text{g/L})^2$ .

Recalculer alors la concentration au fond du récipient. Conclure.

$$\Pi = kT \frac{\phi}{V_p} + 0,01(\rho_p \phi)^2 \quad C_2 = 5.7 \text{ g/L}$$



# Interfacial phenomena



On filtre la même dispersion sur une membrane tubulaire

diamètre 6 mm Longueur 1,2 m  
Vitesse tangentielle 0,05-0,1 m/s

Re=  
360-720

Couche limite

Physico-chimie

Sc=xxx

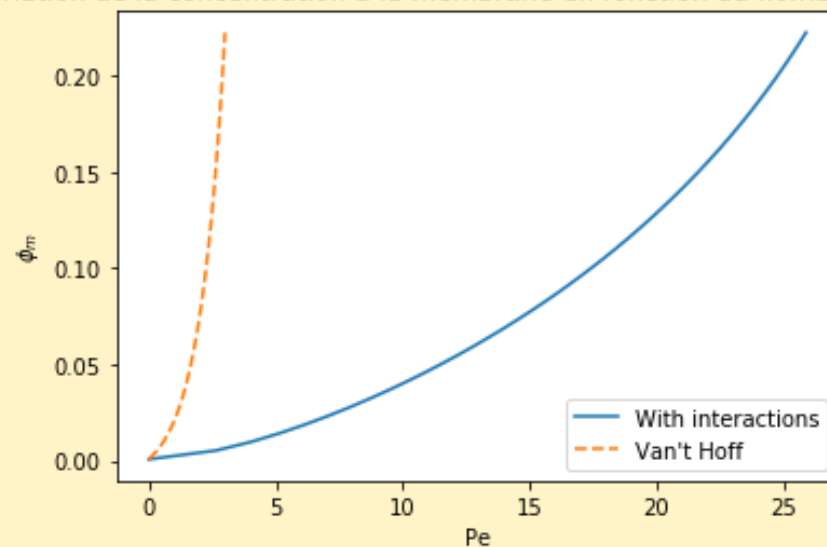
Péclet

Flux de filtration

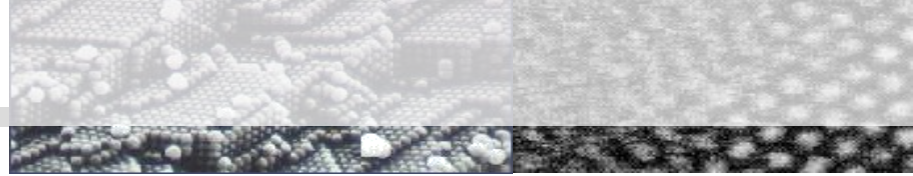
$$Pe = \frac{V_p}{kT} \int_{\Pi_b}^{\Pi_m} \frac{K(\phi)}{\phi} d\Pi$$

En présence de répulsions, la dispersion résiste mieux à la compression induite par la force de traînée liée à la filtration : l'accumulation est moins importante

Variation de la concentration à la membrane en fonction du nombre de Péclet



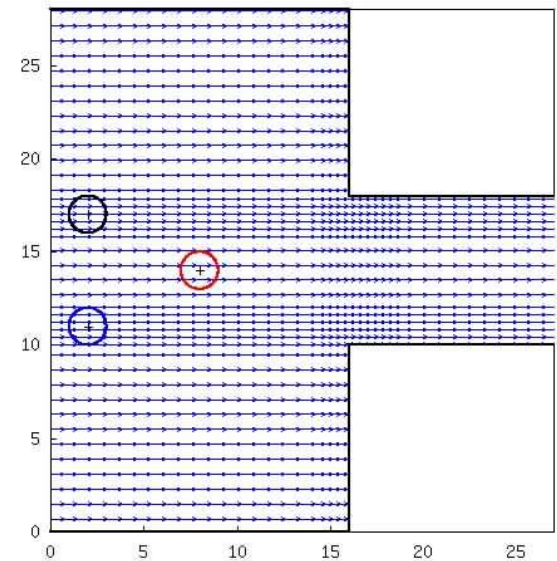
Ce type de modèle peut permettre de déterminer quand un gel va se former à la surface d'une membrane ou d'une goutte qui sèche



## □ Exemples :

□ at hollow fiber scale

□ at pore scale



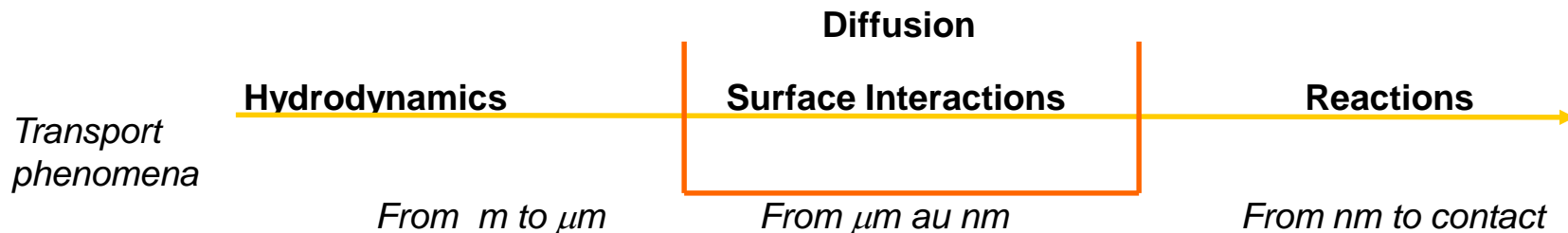
*... Important areas of physical chemistry such as interfacial phenomena, colloids, clusters and, more generally, De Gennes “soft matter” should be revisited using the system approach and chemical engineering methods.*

Jacques Villiermaux, Future challenges for basic research in chemical engineering  
Chemical Engineering Science,48 (1993)

### **Physico-chemical hydrodynamics of colloids**

can - open problems in processes  
- be the source of new processes

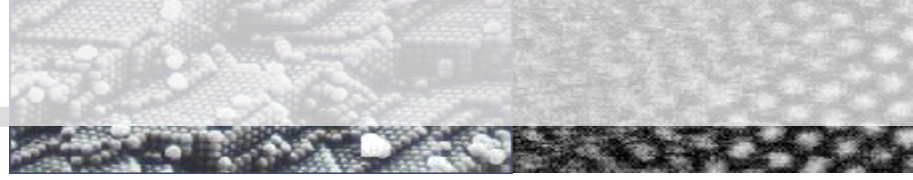
when surface **interactions** are controlled (and well known)...

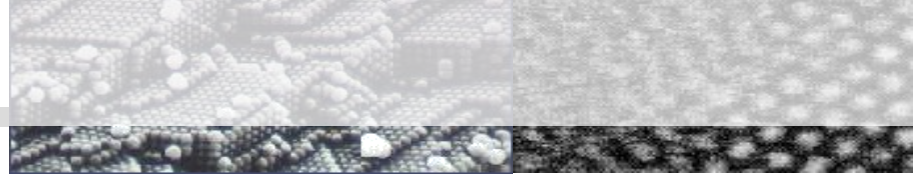




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# *Interfacial phenomena*





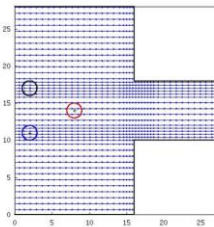
Visite virtuelle



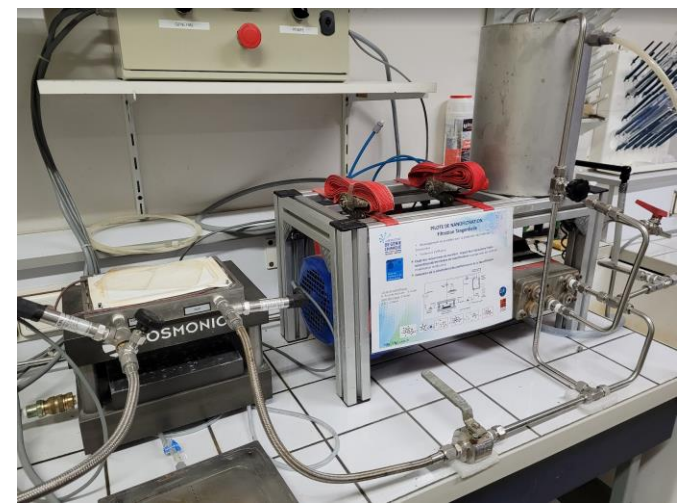


## Filtration à multi-échelle

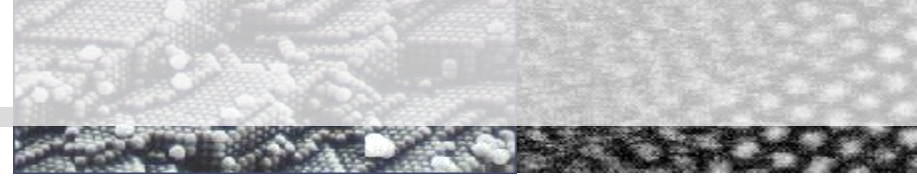
De l'échelle d'un pore



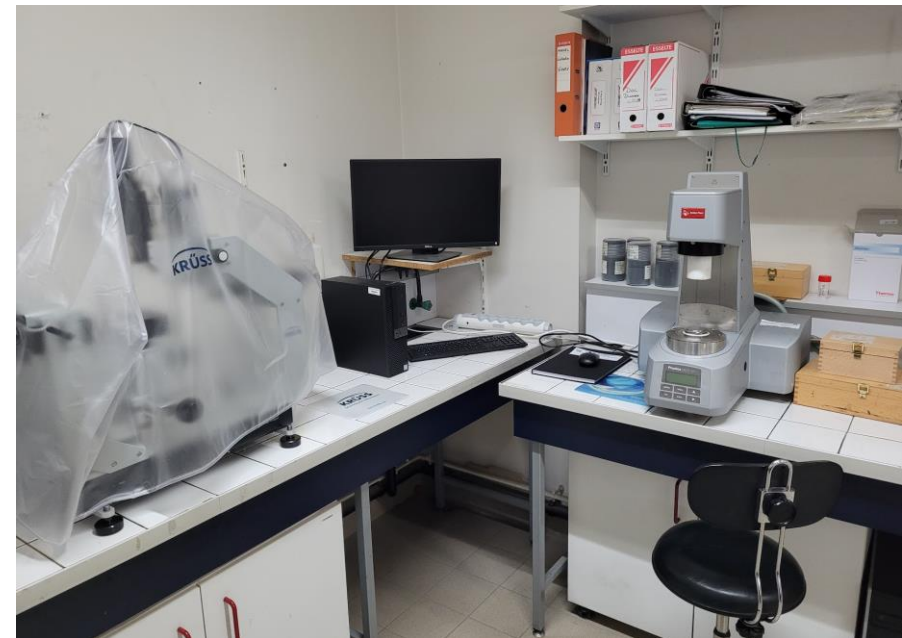
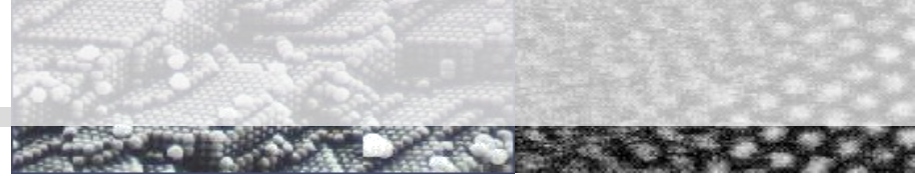
Au pilote



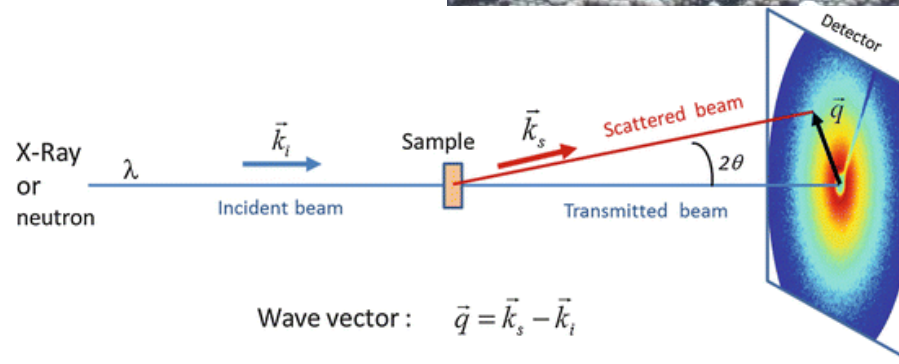
# Interfacial phenomena



# *Interfacial phenomena*

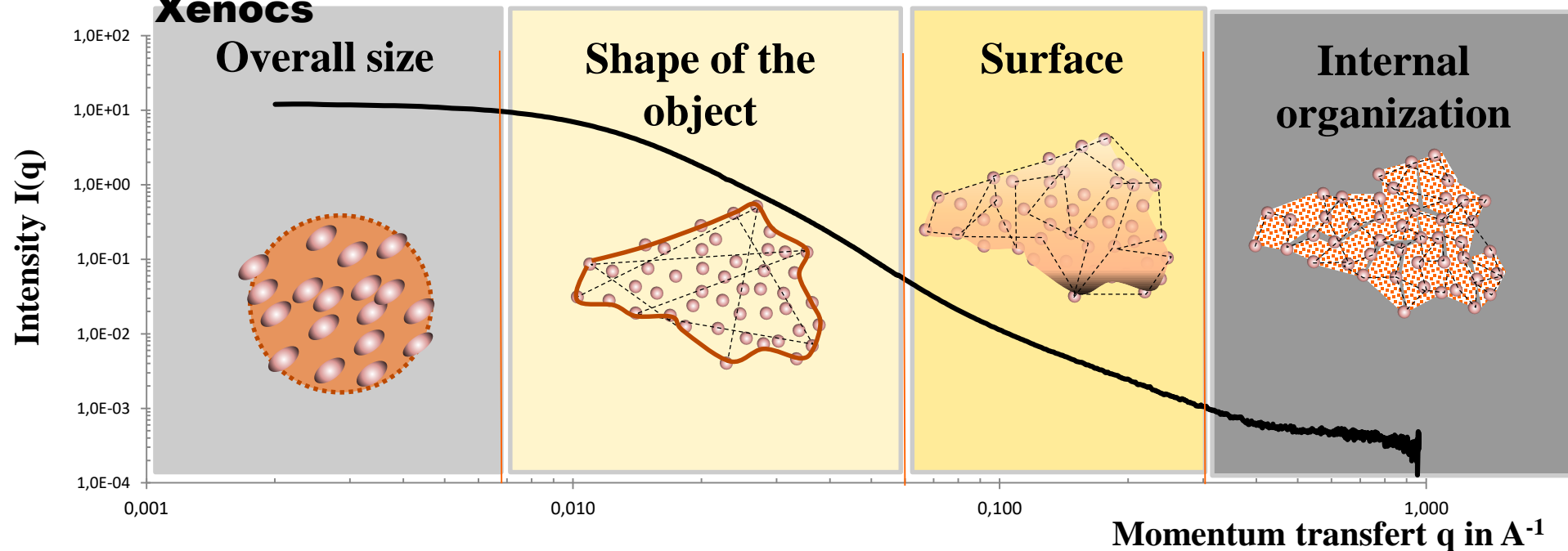


# SAXS



$$q = \frac{4\pi}{\lambda} \sin \frac{\theta}{2}$$

## From Xenocs



Sonde l'organisation de la matière sur une grande gamme d'échelle de taille jusqu'à la centaine de nm



Analyse et modélisation des courbes de diffraction délicate

